Solutions:
Grade 8 Mathematics
Chapter 8: Division of Algebraic Expressions

Exercise 8.1

Q1. Write the degree of each of the following polynomials:

(i) $2x^3 + 5x^2 - 7$
(ii) $5x^2 - 3x + 2$
(iii) $2x + x^2 - 8$
(iv) $\frac{1}{2}y^7 - 12y^6 + 48y^5 - 10$
(v) $3x^3 + 1$
(vi) 5
(vii) $20x^3 + 12x^2y^2 + 20$

Solution:

(i) $2x^3 + 5x^2 - 7$
Degre is the highest power of the variable of a polynomial. In the given polynomial highest power is 3. Therefore, the degree of the polynomial is 3.

(ii) $5x^2 - 3x + 2$
Degre is the highest power of the variable of a polynomial. In the given polynomial highest power is 2. Therefore, the degree of the polynomial is 2.

(iii) $2x + x^2 - 8$
Degre is the highest power of the variable of a polynomial. In the given polynomial highest power is 2. Therefore, the degree of the polynomial is 2.

(iv) $\frac{1}{2}y^7 - 12y^6 + 48y^5 - 10$
Degre is the highest power of the variable of a polynomial. In the given polynomial highest power is 7. Therefore, the degree of the polynomial is 7.

(v) $3x^3 + 1$
Degre is the highest power of the variable of a polynomial. In the given polynomial highest power is 3. Therefore, the degree of the polynomial is 3.

(vi) 5
Degre is the highest power of the variable of a polynomial. In the given polynomial there is no variable term. Therefore, the degree of the polynomial is 0.

(vii) $20x^3 + 12x^2y^2 + 20$
Degre is the highest power of the variable of a polynomial. In the given polynomial, the highest power is 4 (2+2=4). Therefore, the degree of the polynomial is 4.

Q2. Which of the following expressions are not polynomial?

(i) $x^2 + 2x^{-2}$

(ii) $\sqrt{a}x + x^2 - x^3$

(iii) $3y^3 - \sqrt{5}y + 9$

(iv) $ax^{1/2} + ax + 9x^2 + 4$

(v) $3x^{-3} + 2x^{-1} + 4x + 5$

Solution:

(i) $x^2 + 2x^{-2}$

A polynomial never has negative or fractional power. In the given expression x^{-2} has negative power. Therefore, it is not a polynomial.

(ii) $\sqrt{a}x + x^2 - x^3$

A polynomial always has positive power. Therefore, the given expression is a polynomial.

(iii) $3y^3 - \sqrt{5}y + 9$

A polynomial always has positive power. Therefore, the given expression is a polynomial.

(iv) $ax^{1/2} + ax + 9x^2 + 4$

A polynomial never has negative or fractional power. In the given expression $x^{1/2}$ has fractional power. Therefore, it is not a polynomial.

(v) $3x^{-3} + 2x^{-1} + 4x + 5$

A polynomial never has negative or fractional power. In the given expression x^{-3} and x^{-1} has negative power. Therefore, it is not polynomial.

Q3. Write each of the following polynomials in the standard form. Also, write their degree:

(i) $x^2 + 3 + 6x + 5x^4$

(ii) $a^2 + 4 + 5a^6$

(iii) $(x^3 - 1)(x^3 - 4)$

(iv) $(y^3 - 2)(y^3 + 11)$

(v) $\left(a^3 - \frac{3}{8}\right)\left(a^3 + \frac{16}{17}\right)$

(vi) $\left(a + \frac{3}{4}\right)\left(a + \frac{4}{3}\right)$

Solution:

(i) $x^2 + 3 + 6x + 5x^4$

A polynomial in the standard form is written in the decreasing or increasing power of the variable.

Standard form of the polynomial: $5x^4 + x^2 + 6x + 3$ or $3 + 6x + x^2 + 5x^4$

Degree is the highest power of the variable in the given expression.

Therefore, degree of the polynomial is: 4

(ii) $a^2 + 4 + 5a^6$

A polynomial in the standard form is written in the decreasing or increasing power of the variable.

Standard form of the polynomial: $5a^6 + a^2 + 4$ or $4 + a^2 + 5a^6$

Degree is the highest power of the variable in the given expression.

Therefore, degree of the polynomial is: 6

(iii) $(x^3 - 1)(x^3 - 4)$

$(x^3 - 1)(x^3 - 4) = x^6 - 5x^3 + 4$

A polynomial in the standard form is written in the decreasing or increasing power of the variable.

Standard form of the polynomial: $x^6 - 5x^3 + 4$ or $4 - 5x^3 + x^6$

Degree is the highest power of the variable in the given expression.

Therefore, degree of the polynomial is: 6

(iv) $(y^3 - 2)(y^3 + 11)$

$(y^3 - 2)(y^3 + 11) = y^6 + 9y^3 - 22$

A polynomial in the standard form is written in the decreasing or increasing power of the variable.

Standard form of the polynomial: $y^6 + 9y^3 - 22$ or $-22 + 9y^3 + y^6$

Degree is the highest power of the variable in the given expression.

Therefore, degree of the polynomial is: 6

(v) $\left(a^3 - \frac{3}{8}\right)\left(a^3 + \frac{16}{17}\right)$

$$\left(a^3 - \frac{3}{8}\right)\left(a^3 + \frac{16}{17}\right) = a^6 + \frac{77a^3}{136} - \frac{6}{17}$$

A polynomial in the standard form is written in the decreasing or increasing power of the variable.

Standard form of the polynomial: $a^6 + \frac{77a^3}{136} - \frac{6}{17}$ or $-\frac{6}{17} + \frac{77a^3}{136} + a^6$

Degree is the highest power of the variable in the given expression.

Therefore, degree of the polynomial is: 6

(vi) $\left(a + \frac{3}{4}\right)\left(a + \frac{4}{3}\right)$

$$\left(a + \frac{3}{4}\right)\left(a + \frac{4}{3}\right) = a^2 + \frac{25a}{12} + 1$$

A polynomial in the standard form is written in the decreasing or increasing power of the variable.

Standard form of the polynomial: $a^2 + \frac{25a}{12} + 1$ or $1 + \frac{25a}{12} + a^2$

Degree is the highest power of the variable in the given expression.

Therefore, degree of the polynomial is: 2

Exercise 8.2

Q1. Divide:

$6x^3y^3z^2$ by $3x^2yz$

Solution:

$\frac{6x^3y^3z^2}{3x^2yz} = \left(\frac{6}{3}x^{3-2}y^{3-1}z^{2-1}\right) = 2xy^2z$ [Using $a^{\text{n}} \div a^{\text{m}} = a^{\text{n}-\text{m}}$]

Q2. Divide:

$15m^3n^3$ by $5m^2n^2$

Solution:

$\frac{15m^3n^3}{5m^2n^2} = \left(\frac{15}{5}m^{3-2}n^{3-2}\right) = 3mn$ [Using $a^n \div a^m = a^{n-m}$]

Q3. Divide:

$24a^3b^3by - 8ab$

Solution:

$\frac{24a^3b^3}{-8ab} = \left(\frac{24}{-8}a^{3-1}b^{3-1}\right) = -3a^2b^2$ [Using $a^n \div a^m = a^{n-m}$]

Q4. Divide:

$-21abc^2$ by $-7abc$

Solution:

$\frac{-21abc^2}{-7abc} = \left(\frac{-21}{-7}a^{1-1}b^{1-1}c^{2-1}\right) = 3a^\circ b^\circ c = 3c$ [Using $a^n \div a^m = a^{n-m}$] and [$a^\circ = 1$]

Q5. Divide:

xyz^2 by $-9xz$

Solution:

$\frac{xyz^2}{-9xz} = \left(\frac{1}{-9}x^{1-1}yz^{2-1}\right) = -\frac{1yz}{9} = -\frac{yx}{9}$ [Using $a^n \div a^m = a^{n-m}$] and [$a^\circ = 1$]

Q6. Divide:

$-72a^4b^5c^8$ by $-9a^2b^2c^3$

Solution:

$$\frac{-72a^4b^5c^8}{-9a^2b^2c^3}$$

$$= \frac{72}{9}a^{4-2}b^{5-2}c^{8-3}$$

$$= 8a^2b^3c^5$$

Q7. Simplify:

$$\frac{16m^3y^2}{4m^2y}$$

Solution:

$\frac{16m^3y^2}{4m^2y} = \left(\frac{16}{4}m^{3-2}y^{2-1}\right) = 4my$ [Using $a^n \div a^m = a^{n-m}$]

Q8. Simplify:

$$\frac{32m^2n^2p^2}{4mnp}$$

Solution:

$\frac{32m^2n^2p^2}{4mnp} = \left(\frac{32}{4}m^{2-1}n^{2-1}p^{2-1}\right) = 8mnp$ [Using $a^n \div a^m = a^{n-m}$]

Exercise 8.3

Q1. Divide:

$x + 2x^2 + 3x^4 - x^5$ by $2x$

Solution:

$\frac{x+2x^2+3x^4-x^5}{2x} = \frac{x}{2x} + \frac{2x^2}{2x} + \frac{3x^4}{2x} - \frac{x^5}{2x} = \frac{1}{2} + x + \frac{3x^3}{2} - \frac{x^4}{2}$ [Using $a^n \div a^m = a^{n-m}$]

Q2. Divide:

$y^4 - 3y^3 + \frac{1}{2}y^2$ by $3y$

Solution:

$\frac{y^4-3y^3+\frac{y^2}{2}}{3y} = \frac{y^4}{3y} - \frac{3y^3}{3y} + \frac{y^2}{6y} = \frac{y^3}{3} - y^2 + \frac{y}{6}$ [Using $a^n \div a^m = a^{n-m}$]

Q3. Divide:

$-4a^3 + 4a^2 + a$ by $2a$

Solution:

$-\frac{4a^3}{2a} + \frac{4a^2}{2a} + \frac{a}{2a} = -2a^2 + 2a + \frac{1}{2} =$ [Using $a^n \div a^m = a^{n-m}$]

Q4. Divide:

$-x^6 + 2x^4 + 4x^3 + 2x^2$ by $\sqrt{2}x^2$

Solution:

$-\frac{x^6}{\sqrt{2}x^2} + \frac{2x^4}{\sqrt{2}x^2} + \frac{4x^3}{\sqrt{2}x^2} + \frac{2x^2}{\sqrt{2}x^2} = -\frac{x^4}{\sqrt{2}} + \frac{2x^2}{\sqrt{2}} + \frac{4x}{\sqrt{2}} + \frac{2}{\sqrt{2}} = -\frac{x^4}{\sqrt{2}} + \sqrt{2}x^2 + 2\sqrt{2}x + \sqrt{2}$
[Using $a^n \div a^m = a^{n-m}$]

Q5. Divide:

$5z^3 - 6z^2 + 7z$ by $2z$

Solution:

$\frac{5z^3}{2z} - \frac{6z^2}{2z} + \frac{7z}{2z} = \frac{5z^2}{2} - 3z + \frac{7}{2}$ [Using $a^n \div a^m = a^{n-m}$]

Q6. Divide:

$\sqrt{3}a^4 + 2\sqrt{3}a^3 + 3^2 - 6a$ by $3a$

Solution:

$\frac{\sqrt{3}a^4}{3a} + \frac{2\sqrt{3}a^3}{3a} + \frac{3^2}{3a} - \frac{6a}{3a} = \frac{\sqrt{3}a^3}{3} + \frac{2\sqrt{3}a^2}{3} + \frac{3}{a} - 2$ [Using $a^n \div a^m = a^{n-m}$]

Exercise 8.4

Q1. Divide:

$5x^3 - 15x^2 + 25x$ by $5x$

Solution:

$\frac{5x^3}{5x} - \frac{15x^2}{5x} + \frac{25x}{5x} = 5x^2 - 3x + 5$ [Using $a^n \div a^m = a^{n-m}$]

Q2. Divide:

$4z^3 + 6z^2 - z$ by $-\frac{1}{2}z$

Solution:

$\frac{2\times4z^3}{-1z} + \frac{2\times6z^2}{-1z} - \frac{2\times z}{-1z} = -8z^2 - 12z + 2$ [Using $a^n \div a^m = a^{n-m}$]

Q3. Divide:

$9x^2y - 6xy + 12xy^2$ by $-\frac{3}{2}xy$

Solution:

$\frac{2\times9x^2y}{-3xy} - \frac{2\times6xy}{-3xy} + \frac{2\times12xy^2}{-3xy} = -6x + 4 - 8y$ [Using $a^n \div a^m = a^{n-m}$]

Q4. Divide:

$3x^3y^2 + 2x^2y + 15xy$ by $3xy$

Solution:

$\frac{3x^3y^2}{3xy} + \frac{2x^2y}{3xy} + \frac{15xy}{3xy} = x^2y + \frac{2x}{3} + 5$ [Using $a^n \div a^m = a^{n-m}$]

Q5. Divide:

$x^2 + 7x + 12$ by $x + 4$

Solution:

$$
\begin{array}{r|llll}
 & x & +3 & & \\
\hline
x+4 & x^2 & +7x & +12 & \\
 & - & & & \\
 & x^2 & +4x & & \\
\hline
 & & 3x & +12 & \\
 & & - & & \\
 & & 3x & +12 & \\
\hline
 & & & 0 &
\end{array}
$$

$3x \quad +12$

Therefore, answer is: $\frac{x^2+7x+12}{x+4} = x + 3$

Q6. Divide:

$4y^2 + 3y + \frac{1}{2}$ by $2y + 1$

Solution:

$$
\begin{array}{r|lllll}
 & 2y & + & 0.5 & & \\
\hline
2y+1 & 4y^2 & + & 3y & + & 0.5 \\
 & -4y^2 & + & -2y & & \\
\hline
 & & & y & + & 0.5 \\
 & & & -y & + & -0.5 \\
\hline
 & & & & & 0
\end{array}
$$

There is no remainder, so the quotient is $2y + 0.5$.

Q7. Divide:

$3x^3 + 4x^2 + 5x + 18$ by $x + 2$

Solution:

$$
\begin{array}{r|lllllll}
 & 3x^2 & - & 2x & + & 9 & & \\
\hline
x+2 & 3x^3 & + & 4x^2 & + & 5x & + & 18 \\
 & -3x^3 & + & -6x^2 & & & & \\
\hline
 & & - & 2x^2 & + & 5x & + & 18 \\
 & & - & +2x^2 & - & +4x & & \\
\hline
 & & & & & 9x & + & 18 \\
 & & & & & -9x & + & -18 \\
\hline
 & & & & & & & 0
\end{array}
$$

Therefore, $\frac{3x^3+4x^2+5x+18}{x+2} = 3x^2 - 2x + 9$

Q8. Divide:

$14x^2 - 53x + 45$ by $7x - 9$

Solution:

$7x - 9)14x^2 - 53x + 45(2x - 5$

$14x^2 - 18x$

$- \quad +$

$-35x + 45$

$-35x + 45$

Q9. Divide:

$-21 + 71x - 31x^2 - 24x^3$ by $3 - 8x$

Solution:

$3x^2 \quad + 5x \quad - 7$

$-8x + 3 \,\big)\, -24x^3 - 31x^2 + 71x - 21$

$-\,{}_{+}24x^3 + {}_{-}9x^2$

$-40x^2 + 71x - 21$

$-\,{}_{+}40x^2 + {}_{-}15x$

$56x - 21$

${}_{-}56x - {}_{+}21$

0

Therefore, $\frac{-21+71x-31x^2-24x^3}{3-8x} = 3x^2 + 5x - 7$

Q10. Divide:

$3y^4 - 3y^3 - 4y^2 - 4y$ by $y^2 - 2y$

Solution:

$$
\begin{array}{r|l}
 & 3y^2 \quad + 3y \quad + 2 \\
\hline
y^2 - 2y & 3y^4 - 3y^3 - 4y^2 - 4y + 0 \\
 & {}_{-}3y^4 - {}_{+}6y^3 \\
\hline
 & 3y^3 - 4y^2 - 4y + 0 \\
 & {}_{-}3y^3 - {}_{+}6y^2 \\
\hline
 & 2y^2 - 4y + 0 \\
 & {}_{-}2y^2 - {}_{+}4y \\
\hline
 & 0
\end{array}
$$

Therefore, $\frac{3y^4-3y^3-4y^2-4y}{y^2-2y} = 3y^2 + 3y + 2$

Q11. Divide:

$2y^5 + 10y^4 + 6y^3 + y^2 + 5y + 3$ by $2y^3 + 1$

Solution:

$$
\begin{array}{r|l}
 & y^2 \quad + 5y \quad + 3 \\
\hline
2y^3 + 1 & 2y^5 + 10y^4 + 6y^3 + y^2 + 5y + 3 \\
 & {}_{-}2y^5 \qquad + {}_{-}y^2 \\
\hline
 & 10y^4 + 6y^3 + 5y + 3 \\
 & {}_{-}10y^4 \qquad + {}_{-}5y \\
\hline
 & 6y^3 + 3 \\
 & {}_{-}6y^3 + {}_{-}3 \\
\hline
 & 0
\end{array}
$$

Therefore, $\frac{2y^5+10y^4+6y^3+y^2+5y+3}{2y^3+1} = y^2 + 5y + 3$

Q12. Divide:

$x^4 - 2x^3 + 2x^2 + x + 4$ by $x^2 + x + 1$

Solution:

$$
\begin{array}{r|l}
 & x^2 - 3x + 4 \\ \hline
x^2 + x + 1 & x^4 - 2x^3 + 2x^2 + x + 4 \\
 & \underset{-}{x^4} + \underset{-}{x^3} + \underset{-}{x^2} \\ \hline
 & -3x^3 + x^2 + x + 4 \\
 & -\underset{+}{3x^3} - \underset{+}{3x^2} - \underset{+}{3x} \\ \hline
 & 4x^2 + 4x + 4 \\
 & \underset{-}{4x^2} + \underset{-}{4x} + \underset{-}{4} \\ \hline
 & 0
\end{array}
$$

Therefore, $\frac{x^4-2x^3+2x^2+x+4}{x^2+x+1} = x^2 - 3x + 4$

Q13. Divide:

$m^3 - 14m^2 + 37m - 26$ by $m^2 - 12m + 13$

Solution:

$$
\begin{array}{r|l}
 & m - 2 \\ \hline
m^2 - 12m + 13 & m^3 - 14m^2 + 37m - 26 \\
 & \underset{-}{m^3} - \underset{+}{12m^2} + \underset{-}{13m} \\ \hline
 & -2m^2 + 24m - 26 \\
 & -\underset{+}{2m^2} + \underset{-}{24m} - \underset{+}{26} \\ \hline
 & 0
\end{array}
$$

Therefore, $\frac{m^3-14m^2+37m-26}{m^2-12m+13} = m - 2$

Q14. Divide:

$x^4 + x^2 + 1$ by $x^2 + x + 1$

Solution:

$$
\begin{array}{r|llllllllll}
 & x^2 & - & x & & + & 1 & & & & \\
\hline
x^2+x+1 & x^4 & + & 0x^3 & + & x^2 & + & 0x & + & 1 \\
 & {}_{-}x^4 & + & {}_{-}x^3 & + & {}_{-}x^2 & & & & \\
\hline
 & & - & x^3 & & & + & 0x & + & 1 \\
 & & - & {}_{+}x^3 & - & {}_{+}x^2 & - & {}_{+}x & & \\
\hline
 & & & & & x^2 & + & x & + & 1 \\
 & & & & & {}_{-}x^2 & + & {}_{-}x & + & {}_{-}1 \\
\hline
 & & & & & & & & & 0
\end{array}
$$

Therefore, $\frac{x^4+x^2+1}{x^2+x+1} = x^2 - x + 1$

Q15. Divide:

$x^5 + x^4 + x^3 + x^2 + x + 1$ by $x^3 + 1$

Solution:

$$
\begin{array}{r|lllllllllll}
 & x^2 & + & x & & + & 1 & & & & & \\
\hline
x^3+1 & x^5 & + & x^4 & + & x^3 & + & x^2 & + & x & + & 1 \\
 & {}_{-}x^5 & & & & & + & {}_{-}x^2 & & & & \\
\hline
 & & & x^4 & + & x^3 & & & + & x & + & 1 \\
 & & & {}_{-}x^4 & & & & & + & {}_{-}x & & \\
\hline
 & & & & & x^3 & & & & & + & 1 \\
 & & & & & {}_{-}x^3 & & & & & + & {}_{-}1 \\
\hline
 & & & & & & & & & & & 0
\end{array}
$$

Therefore, $\frac{x^5+x^4+x^3+x^2+x+1}{x^3+1} = x^2 + x + 1$

Q16. Divide each of the following and find the quotient and remainder:

$14x^3 - 5x^2 + 9x - 1$ by $2x - 1$

Solution:

$$
\begin{array}{r|llll}
 & 7x^2 & +\ x & +\ 5 & \\
\hline
2x-1 & 14x^3 - & 5x^2 + & 9x - & 1 \\
 & {}_{-}14x^3 - & {}_{+}7x^2 & & \\
\hline
 & & 2x^2 + & 9x - & 1 \\
 & & {}_{-}2x^2 - & {}_{+}x & \\
\hline
 & & & 10x - & 1 \\
 & & & {}_{-}10x - & {}_{+}5 \\
\hline
 & & & & 4
\end{array}
$$

Therefore,
Quotient: $7x^2 + x + 5$
Remainder: 4

Q17. Divide each of the following and find the quotient and remainder:
$3x^3 - x^2 - 10x - 3$ by $x - 3$

Solution:

$$
\begin{array}{r|llll}
 & 3x^2 & +\ 8x & +\ 14 & \\
\hline
x-3 & 3x^3 - & x^2 - & 10x - & 3 \\
 & {}_{-}3x^3 - & {}_{+}9x^2 & & \\
\hline
 & & 8x^2 - & 10x - & 3 \\
 & & {}_{-}8x^2 - & {}_{+}24x & \\
\hline
 & & & 14x - & 3 \\
 & & & {}_{-}14x - & {}_{+}42 \\
\hline
 & & & & 39
\end{array}
$$

Therefore,
Quotient: $3x^2 + 8x + 14$
Remainder: 39

Q18. Divide each of the following and find the quotient and remainder:
$6x^3 + 11x^2 - 39x - 65$ by $3x^2 + 13x + 13$

Solution:

$$
\begin{array}{r|llllllll}
 & 2x & - & 5 & & & & & \\
\hline
3x^2 + 13x + 13 & 6x^3 & + & 11x^2 & - & 39x & - & 65 \\
 & _{-}6x^3 & + & _{-}26x^2 & + & _{-}26x & & \\
\hline
 & & - & 15x^2 & - & 65x & - & 65 \\
 & & - & _{+}15x^2 & - & _{+}65x & - & _{+}65 \\
\hline
 & & & & & & & 0
\end{array}
$$

Therefore,
Quotient: $2x - 5$
Remainder: 0

Q19. Divide each of the following and find the quotient and remainder:
$30x^4 + 11x^3 - 82x^2 - 12x + 48$ by $3x^2 + 2x - 4$

Solution:

$$
\begin{array}{r|llllllllll}
 & 10x^2 & - & 3x & - & 12 & & & & \\
\hline
3x^2 + 2x - 4 & 30x^4 & + & 11x^3 & - & 82x^2 & - & 12x & + & 48 \\
 & _{-}30x^4 & + & _{-}20x^3 & - & _{+}40x^2 & & & & \\
\hline
 & & - & 9x^3 & - & 42x^2 & - & 12x & + & 48 \\
 & & - & _{+}9x^3 & - & _{+}6x^2 & + & _{-}12x & & \\
\hline
 & & & & - & 36x^2 & - & 24x & + & 48 \\
 & & & & - & _{+}36x^2 & - & _{+}24x & + & _{-}48 \\
\hline
 & & & & & & & & & 0
\end{array}
$$

Therefore,
Quotient: $10x^2 - 3x - 12$
Remainder: 0

Q20. Divide each of the following and find the quotient and remainder:
$x^4 - 4x^2 + 4$ by $3x^2 - 4x + 2$

Solution:

$$
\begin{array}{r|l}
 & 3x^2 \quad + 4x \quad + 2 \\
\hline
3x^2 - 4x + 2 & 9x^4 + 0x^3 - 4x^2 + 0x + 4 \\
 & {}_{-}9x^4 \; - \; {}_{+}12x^3 \; + \; {}_{-}6x^2 \\
\hline
 & \qquad 12x^3 - 10x^2 + 0x + 4 \\
 & \qquad {}_{-}12x^3 \; - \; {}_{+}16x^2 \; + \; {}_{-}8x \\
\hline
 & \qquad\qquad 6x^2 - 8x + 4 \\
 & \qquad\qquad {}_{-}6x^2 \; - \; {}_{+}8x \; + \; {}_{-}4 \\
\hline
 & \qquad\qquad\qquad 0
\end{array}
$$

Therefore,
Quotient: $3x^2 + 4x + 2$
Remainder: 0

Q21. Verify division algorithm i.e. Dividend = Divisor × Quotient + Remainder, in each of the following. Also, write the quotient and remainder.

(i) $14x^2 + 13x - 15$ by $7x - 4$
(ii) $15z^3 - 20z^2 + 13z - 12$ by $3z - 6$
(iii) $6y^5 - 28y^3 + 3y^2 + 30y - 9$ by $2y^2 - 6$
(iv) $34x - 22x^3 - 12x^4 - 10x^2 - 75$ by $3x + 7$
(v) $15y^4 - 16y^3 + 9y^2 - \frac{10}{3}y + 6$ by $3y - 2$
(vi) $4y^3 + 8y + 8y^2 + 7$ by $2y^2 - y + 1$
(vii) $6y^5 + 4y^4 + 4y^3 + 7y^2 + 27y + 6$ by $2y^3 + 1$

Solution:

(i)

$$
\begin{array}{r|l}
 & 2x \quad + 3 \\
\hline
7x - 4 & 14x^2 + 13x - 15 \\
 & {}_{-}14x^2 \; - \; {}_{+}8x \\
\hline
 & \qquad 21x - 15 \\
 & \qquad {}_{-}21x \; - \; {}_{+}12 \\
\hline
 & \qquad\qquad - 3
\end{array}
$$

Therefore,
Quotient: $2x + 3$
Remainder: -3
Now,
Dividend = Divisor × Quotient + Remainder

$$14x^2 + 13x - 15 = (7x - 4) \times (2x + 3) + (-3)$$
$$14x^2 + 13x - 15 = 14x^2 + 21x - 8x - 12 - 3$$
$$14x^2 + 13x - 15 = 14x^2 + 13x - 15$$

(ii)

$5z^2 + 3.33z + 11$

$3z - 6 \overline{)\ 15z^3 - 20z^2 + 13z - 12}$

${}_{-}15z^3 - {}_{+}30z^2$

$10z^2 + 13z - 12$

${}_{-}10z^2 - {}_{+}20z$

$33z - 12$

${}_{-}33z - {}_{+}66$

54

Therefore,

Quotient: $5z^2 + 3.33z + 11$

Remainder: 54

Now,

Dividend = Divisor × Quotient + Remainder

$$15z^3 - 20z^2 + 13z - 12 = (3z - 6) \times \left(5z^2 + \frac{10z}{3} + 11\right) + 54$$
$$15z^3 - 20z^2 + 13z - 12 = 15z^3 + 10z^2 + 33z - 30z^2 - 20z + 54$$
$$15z^3 - 20z^2 + 13z - 12 = 15z^3 - 20z^2 + 13z - 12$$

(iii)

$$
\begin{array}{r|l}
 & 3y^3 + 0y^2 - 5y + 1.5 \\
\hline
2y^2 - 6 & 6y^5 + 0y^4 - 28y^3 + 3y^2 + 30y - 9 \\
 & {}_{-}6y^5 \quad - {}_{+}18y^3 \\
\hline
 & 0y^4 - 10y^3 + 3y^2 + 30y - 9 \\
 & {}_{-}0y^4 \quad - {}_{+}0y^2 \\
\hline
 & - 10y^3 + 3y^2 + 30y - 9 \\
 & - {}_{+}10y^3 \quad + {}_{-}30y \\
\hline
 & 3y^2 - 9 \\
 & {}_{-}3y^2 \quad - {}_{+}9 \\
\hline
 & 0
\end{array}
$$

Therefore,

Quotient: $3y^3 - 5y + \frac{3}{2}$

Remainder: 0

Now,

Dividend = Divisor × Quotient + Remainder

$$6y^5 - 28y^3 + 3y^2 + 30y - 9 = (2y^2 - 6) \times \left(3y^3 - 5y + \frac{3}{2}\right) + 0$$

$$6y^5 - 28y^3 + 3y^2 + 30y - 9 = 6y^5 - 10y^3 + 3y^2 - 18y^3 + 30y - 9$$

$$6y^5 - 28y^3 + 3y^2 + 30y - 9 = 6y^5 - 28y^3 + 3y^2 + 30y - 9$$

(iv)

$$
\begin{array}{r|l}
 & - 4x^3 + 2x^2 - 8x + 30 \\
\hline
3x + 7 & - 12x^4 - 22x^3 - 10x^2 + 34x - 75 \\
 & - {}_{+}12x^4 - {}_{+}28x^3 \\
\hline
 & 6x^3 - 10x^2 + 34x - 75 \\
 & {}_{-}6x^3 + {}_{-}14x^2 \\
\hline
 & - 24x^2 + 34x - 75 \\
 & - {}_{+}24x^2 - {}_{+}56x \\
\hline
 & 90x - 75 \\
 & {}_{-}90x + {}_{-}210 \\
\hline
 & - 285
\end{array}
$$

Therefore,

Quotient: $-4x^3 + 2x^2 - 8x + 30$

Remainder: -285

Now,

Dividend = Divisor × Quotient + Remainder

$-12x^4 - 22x^3 - 10x^2 + 34x - 75 = (3x + 7) \times (-4x^3 + 2x^2 - 8x + 30) - 285$

$-12x^4 - 22x^3 - 10x^2 + 34x - 75$

$= -12x^4 + 6x^3 - 24x^2 - 28x^3 + 14x^2 + 90x - 56x + 210 - 285$

$-12x^4 - 22x^3 - 10x^2 + 34x - 75 = -12x^4 - 22x^3 - 10x^2 + 34x - 75$

(v)

$$
\begin{array}{r|lllll}
 & 5y^3 & -\ 2y^2 & +\ 1.67y & +\ 0 & \\
\hline
3y - 2 & 15y^4 & -\ 16y^3 & +\ 9y^2 & -\ 3.33y & +\ 6 \\
 & {}_{-}15y^4 & -\ {}_{+}10y^3 & & & \\
\hline
 & & -\ 6y^3 & +\ 9y^2 & -\ 3.33y & +\ 6 \\
 & & -\ {}_{+}6y^3 & +\ {}_{-}4y^2 & & \\
\hline
 & & & 5y^2 & -\ 3.33y & +\ 6 \\
 & & & {}_{-}5y^2 & -\ {}_{+}3.33y & \\
\hline
 & & & & 0y & +\ 6 \\
 & & & & {}_{-}0y & -\ {}_{+}0 \\
\hline
 & & & & & 6
\end{array}
$$

Therefore,

Quotient: $5y^3 - 2y^2 + \frac{5y}{3}$

Remainder: 6

Now,

Dividend = Divisor × Quotient + Remainder

$15y^4 - 16y^3 + 9y^2 - \dfrac{10y}{3} + 6 = (3y - 2) \times \left(5y^3 - 2y^2 + \dfrac{5y}{3}\right) + 6$

$15y^4 - 16y^3 + 9y^2 - \dfrac{10y}{3} + 6 = 15y^4 - 6y^3 + 5y^2 - 10y^3 + 4y^2 - \dfrac{10y}{3} + 6$

$15y^4 - 16y^3 + 9y^2 - \dfrac{10y}{3} + 6 = 15y^4 - 16y^3 + 9y^2 - \dfrac{10y}{3} + 6$

(vi)

$$\begin{array}{r|l} & 2y \quad + \; 5 \\ \hline 2y^2 - y + 1 & 4y^3 \; + \; 8y^2 \; + \; 8y \; + \; 7 \\ & _{-}4y^3 \; - \; _{+}2y^2 \; + \; _{-}2y \\ \hline & \qquad 10y^2 \; + \; 6y \; + \; 7 \\ & \qquad _{-}10y^2 \; - \; _{+}5y \; + \; _{-}5 \\ \hline & \qquad\qquad 11y \; + \; 2 \end{array}$$

Therefore,
Quotient: $2y + 5$
Remainder: $11y + 2$
Now,
Dividend = Divisor × Quotient + Remainder
$4y^3 + 8y^2 + 8y + 7 = (2y^2 - y + 1) \times (2y + 5) + 11y + 2$
$4y^3 + 8y^2 + 8y + 7 = 4y^3 + 10y^2 - 2y^2 - 5y + 2y + 5 + 11y + 2$
$4y^3 + 8y^2 + 8y + 7 = 4y^3 + 8y^2 + 8y + 7$

(vii)

$$\begin{array}{r|l} & 3y^2 \; + \; 2y \; + \; 2 \\ \hline 2y^3 + 1 & 6y^5 \; + \; 4y^4 \; + \; 4y^3 \; + \; 7y^2 \; + \; 27y \; + \; 6 \\ & _{-}6y^5 \qquad\qquad\qquad + \; _{-}3y^2 \\ \hline & \qquad 4y^4 \; + \; 4y^3 \; + \; 4y^2 \; + \; 27y \; + \; 6 \\ & \qquad _{-}4y^4 \qquad\qquad\qquad + \; _{-}2y \\ \hline & \qquad\qquad 4y^3 \; + \; 4y^2 \; + \; 25y \; + \; 6 \\ & \qquad\qquad _{-}4y^3 \qquad\qquad\qquad + \; _{-}2 \\ \hline & \qquad\qquad\qquad 4y^2 \; + \; 25y \; + \; 4 \end{array}$$

Therefore,
Quotient: $3y^2 + 2y + 2$
Remainder: $4y^2 + 25y + 4$
Now,
Dividend = Divisor × Quotient + Remainder
$6y^5 + 4y^4 + 4y^3 + 7y^2 + 27y + 6 = (2y^3 + 1) \times (3y^2 + 2y + 2) + 4y^2 + 25y + 4$

$$6y^5 + 4y^4 + 4y^3 + 7y^2 + 27y + 6$$
$$= 6y^5 + 4y^4 + 4y^3 + 3y^2 + 2y + 2 + 4y^2 + 25y + 4$$
$$6y^5 + 4y^4 + 4y^3 + 7y^2 + 27y + 6 = 6y^5 + 4y^4 + 4y^3 + 7y^2 + 27y + 6$$

Q22. Divide $15y^4 + 16y^3 + \frac{10}{3}y - 9y^2 - 6$ by $3y - 2$. Write down the coeficients of the terms in the quotient.

Solution:

	$5y^3$	$+ 8.67y^2$	$+ 2.78y$	$+ 2.96$
$3y - 2$	$15y^4 + 16y^3$	$- 9y^2$	$+ 3.33y$	$- 6$
	$_-15y^4 - {}_+10y^3$			
	$26y^3$	$- 9y^2$	$+ 3.33y$	$- 6$
	$_-26y^3$	$- {}_+17.33y^2$		
		$8.33y^2$	$+ 3.33y$	$- 6$
		$_-8.33y^2$	$- {}_+5.56y$	
			$8.89y$	$- 6$
			$_-8.89y$	$- {}_+5.93$
				$- 0.07$

Therefore,

Quotient: $5y^3 + 8.67y^2 + 2.78y + 2.96$ or $5y^3 + \frac{26}{3}y^2 + \frac{25}{9}y + \frac{80}{27}$

Remainder: $-\frac{2}{27}$ or -0.07

Coefficient of $y^3 = 5$; Coefficient of $y^2 = \frac{26}{3}$; Coefficient of $y = \frac{25}{9}$; Constant term $= \frac{80}{27}$

Q23. Using division of polynomials state whether:

(i) $x + 6$ is a factor of $x^2 - x - 42$

(ii) $4x - 1$ is a factor of $4x^2 - 13x - 12$

(iii) $2y - 5$ is a factor of $4y^4 - 10y^3 - 10y^2 + 30y - 15$

(iv) $3y^2 + 5$ is a factor of $6y^5 + 15y^4 + 16y^3 + 4y^2 + 10y - 35$

(v) $z^2 + 3$ is a factor of $z^5 - 9z$

(vi) $2x^2 - x + 3$ is a factor of $6x^5 - x^4 + 4x^3 - 5x^2 - x - 15$

Solution:

(i) $x + 6$ is a factor of $x^2 - x - 42$

$$
\begin{array}{r|l}
 & x - 7 \\
\hline
x+6 & x^2 - x - 42 \\
 & \underline{-x^2 + -6x} \\
 & \quad -7x - 42 \\
 & \quad \underline{-+7x - +42} \\
 & \qquad 0
\end{array}
$$

Since remainder is 0 therefore $(x+6)$ is a factor of $x^2 - x - 42$.

(ii) $4x - 1$ is a factor of $4x^2 - 13x - 12$

$$
\begin{array}{r|l}
 & x - 3 \\
\hline
4x-1 & 4x^2 - 13x - 12 \\
 & \underline{-4x^2 - +x} \\
 & \quad -12x - 12 \\
 & \quad \underline{-+12x + -3} \\
 & \qquad -15
\end{array}
$$

Therefore,

Quotient: $x - 3$

Remainder: -15

Since remainder is -15, therefore $(4x - 1)$ is not a factor of $4x^2 - 13x - 12$.

(iii) $2y - 5$ is a factor of $4y^4 - 10y^3 - 10y^2 + 30y - 15$

$$
\begin{array}{r|l}
 & 2y^3 - 5y + 2.5 \\
\hline
2y-5 & 4y^4 - 10y^3 - 10y^2 + 30y - 15 \\
 & \underline{-4y^4 - +10y^3} \\
 & \quad -10y^2 + 30y - 15 \\
 & \quad \underline{-+10y^2 + -25y} \\
 & \qquad 5y - 15 \\
 & \qquad \underline{-5y - +12.5} \\
 & \qquad\quad -2.5
\end{array}
$$

Therefore,

Quotient: $2y^3 - 5y + \frac{5}{2}$

Remainder: -2.5 or $-\frac{5}{2}$

Since remainder is $-\frac{5}{2}$ therefore $(2y - 5)$ is not a factor of $4y^4 - 10y^3 - 10y^2 + 30y - 15$.

(iv) $3y^2+5$ is a factor of $6y^5+15y^4+16y^3+4y^2+10y-35$

	$2y^3$	$+\ 5y^2$	$+\ 2y$	$-\ 7$		
$3y^2+5$	$6y^5$	$+\ 15y^4$	$+\ 16y^3$	$+\ 4y^2$	$+\ 10y$	$-\ 35$
	$_{-}6y^5$		$+\ _{-}10y^3$			
		$15y^4$	$+\ 6y^3$	$+\ 4y^2$	$+\ 10y$	$-\ 35$
		$_{-}15y^4$		$+\ _{-}25y^2$		
			$6y^3$	$-\ 21y^2$	$+\ 10y$	$-\ 35$
			$_{-}6y^3$		$+\ _{-}10y$	
				$-\ 21y^2$		$-\ 35$
				$-\ _{+}21y^2$		$-\ _{+}35$
						0

Since remainder is 0 therefore $(3y^2+5)$ is a factor of $6y^5+15y^4+16y^3+4y^2+10y-35$.

(v) z^2+3 is a factor of z^5-9z

	z^3	$+\ 0z^2$	$-\ 3z$			
z^2+3	z^5	$+\ 0z^4$	$+\ 0z^3$	$+\ 0z^2$	$-\ 9z$	$+\ 0$
	$_{-}z^5$		$+\ _{-}3z^3$			
		$0z^4$	$-\ 3z^3$	$+\ 0z^2$	$-\ 9z$	$+\ 0$
		$_{-}0z^4$		$+\ _{-}0z^2$		
			$-\ 3z^3$		$-\ 9z$	$+\ 0$
			$-\ _{+}3z^3$		$-\ _{+}9z$	
						0

Quotient: z^3-3z
Remainder: 0
Since remainder is 0 therefore (z^3+3) is a factor of z^5-9z.

(vi) $2x^2-x+3$ is a factor of $6x^5-x^4+4x^3-5x^2-x-15$

	$3x^3$	$+ x^2$	$- 2x$	$- 5$			
$2x^2 - x + 3$	$6x^5$	$- x^4$	$+ 4x^3$	$- 5x^2$	$- x$	$- 15$	
	$_{-}6x^5$	$- \, _{+}3x^4$	$+ \, _{-}9x^3$				
		$2x^4$	$- 5x^3$	$- 5x^2$	$- x$	$- 15$	
		$_{-}2x^4$	$- \, _{+}x^3$	$+ \, _{-}3x^2$			
			$- 4x^3$	$- 8x^2$	$- x$	$- 15$	
			$- \, _{+}4x^3$	$+ \, _{-}2x^2$	$- \, _{+}6x$		
				$- 10x^2$	$+ 5x$	$- 15$	
				$- \, _{+}10x^2$	$+ \, _{-}5x$	$- \, _{+}15$	
						0	

Quotient: $3x^3 + x^2 - 2x - 5$

Remainder: 0

Since remainder is 0 therefore $(2x^2 - x + 3)$ is a factor of $6x^5 - x^4 + 4x^3 - 5x^2 - x - 15$.

Q24. Find the value of a, if $x + 2$ is a factor of $4x^4 + 2x^3 - 3x^2 + 8x + 5a$.

Solution:

$x + 2$ is a factor of $4x^4 + 2x^3 - 3x^2 + 8x + 5a$.

$x + 2 = 0$

$x = -2$

Therefore, substitute $x = -2$ in the given equation we get,

$4(-2)^4 + 2(-2)^3 - 3(-2)^2 + 8(-2) + 5a = 0$

$64 - 16 - 12 - 16 + 5a = 0$

$20 + 5a = 0$

$5a = -20$

$a = -\frac{20}{5}$

$a = -4$

Q25. What must be added to $x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polymonial is exactly divible by $x^2 + 2x - 3$.

Solution:

$$\begin{array}{r|l} & x^2 \qquad\qquad + 1 \\ \hline x^2 + 2x - 3 & x^4 + 2x^3 - 2x^2 + x - 1 \\ & \underset{-}{}x^4 + \underset{-}{}2x^3 - \underset{+}{}3x^2 \\ \hline & \qquad\qquad x^2 + x - 1 \\ & \qquad\qquad \underset{-}{}x^2 + \underset{-}{}2x - \underset{+}{}3 \\ \hline & \qquad\qquad\qquad -x + 2 \end{array}$$

Quotient: $x^2 + 1$

Remainder: $-x + 2$

Therefore, the required term to be added is $x - 2$.

Exercise 8.5

Q1. Divide the first polynomial by the second polynomial in each of the following. Also write the quotient and remainder:

(i) $3x^2 + 4x + 5, x - 2$

(ii) $10x^2 - 7x + 8, 5x - 3$

(iii) $5y^3 - 6y^2 + 6y - 1, 5y - 1$

(iv) $x^4 - x^3 + 5x, x - 1$

(v) $y^4 - y^2, y^2 - 2$

Solution:

(i) $3x^2 + 4x + 5, x - 2$

$$\begin{array}{r|l} & 3x + 10 \\ \hline x - 2 & 3x^2 + 4x + 5 \\ & \underset{-}{}3x^2 - \underset{+}{}6x \\ \hline & \qquad 10x + 5 \\ & \qquad \underset{-}{}10x - \underset{+}{}20 \\ \hline & \qquad\qquad 25 \end{array}$$

Therefore,

Quotient: $3x + 10$

Remainder: 25

(ii) $10x^2 - 7x + 8, 5x - 3$

$$
\begin{array}{r|l}
 & 2x \quad - 0.2 \\
\hline
5x - 3 & 10x^2 - 7x + 8 \\
 & \underset{-}{}10x^2 \; \underset{+}{-}\,6x \\
\hline
 & \qquad -x + 8 \\
 & \qquad \underset{+}{-}\,x \; \underset{-}{+}\,0.6 \\
\hline
 & \qquad\qquad 7.4
\end{array}
$$

Quotient: $2x - 0.2$ or $2x - \frac{1}{5}$

Remainder: 7.4 or $\frac{37}{5}$

(iii) $5y^3 - 6y^2 + 6y - 1, 5y - 1$

$$
\begin{array}{r|l}
 & y^2 \quad - y \quad + 1 \\
\hline
5y - 1 & 5y^3 - 6y^2 + 6y - 1 \\
 & \underset{-}{}5y^3 \; \underset{+}{-}\,y^2 \\
\hline
 & \qquad -5y^2 + 6y - 1 \\
 & \qquad \underset{+}{-}\,5y^2 \; \underset{-}{+}\,y \\
\hline
 & \qquad\qquad 5y - 1 \\
 & \qquad\qquad \underset{-}{}5y \; \underset{+}{-}\,1 \\
\hline
 & \qquad\qquad\qquad 0
\end{array}
$$

Therefore,
Quotient: $y^2 - y + 1$
Remainder: 0

(iv) $x^4 - x^3 + 5x, x - 1$

$$
\begin{array}{r|l}
 & x^3 \qquad + 0x \quad + 5 \\
\hline
x - 1 & x^4 - x^3 + 0x^2 + 5x + 0 \\
 & \underset{-}{}x^4 \; \underset{+}{-}\,x^3 \\
\hline
 & \qquad\quad 0x^2 + 5x + 0 \\
 & \qquad\quad \underset{-}{}0x^2 \; \underset{+}{-}\,0x \\
\hline
 & \qquad\qquad\quad 5x + 0 \\
 & \qquad\qquad\quad \underset{-}{}5x \; \underset{+}{-}\,5 \\
\hline
 & \qquad\qquad\qquad\quad 5
\end{array}
$$

Therefore,
Quotient: $x^3 + 5$
Remainder: 5

(v) $y^4 - y^2, y^2 - 2$

$$
\begin{array}{r|l}
 & y^2 + 0y + 1 \\
\hline
y^2 - 2 & y^4 + 0y^3 - y^2 + 0y + 0 \\
 & \underset{-}{}y^4 \quad \underset{+}{-} 2y^2 \\
\hline
 & 0y^3 + y^2 + 0y + 0 \\
 & \underset{-}{}0y^3 \quad \underset{+}{-} 0y \\
\hline
 & y^2 + 0 \\
 & \underset{-}{}y^2 \quad \underset{+}{-} 2 \\
\hline
 & 2
\end{array}
$$

Therefore,
Quotient: $y^2 + 1$
Remainder: 2

Q2. Find whether the first polynomial is a factor of the second polynomial:
(i) $x + 1, 2x^2 + 5x + 4$
(ii) $y - 2, 3y^3 + 5y^2 + 5y + 2$
(iii) $4x^2 - 5, 4x^4 + 7x^2 + 15$
(iv) $4 - z, 3z^2 - 13z + 4$
(v) $2a - 3, 10a^2 - 9a - 5$
(vi) $4y + 1, 8y^2 - 2y + 1$

Solution:

(i) $x + 1, 2x^2 + 5x + 4$

$$
\begin{array}{r|l}
 & 2x + 3 \\
\hline
x + 1 & 2x^2 + 5x + 4 \\
 & \underset{-}{}2x^2 + \underset{-}{}2x \\
\hline
 & 3x + 4 \\
 & \underset{-}{}3x + \underset{-}{}3 \\
\hline
 & 1
\end{array}
$$

Quotient: $2x + 3$
Remainder: 1
Since remainder is 1 therefore the first polynomial is not a factor of the second polynomial.

(ii) $y-2, 3y^3+5y^2+5y+2$

$$
\begin{array}{r|l}
 & 3y^2 + 11y + 27 \\
\hline
y-2 & 3y^3 + 5y^2 + 5y + 2 \\
 & \underline{{}_{-}3y^3 \ {}_{+}6y^2} \\
 & \quad 11y^2 + 5y + 2 \\
 & \quad \underline{{}_{-}11y^2 \ {}_{+}22y} \\
 & \qquad 27y + 2 \\
 & \qquad \underline{{}_{-}27y \ {}_{+}54} \\
 & \qquad\qquad 56
\end{array}
$$

Quotient: $3y^2+11y+27$
Remainder: 56
Since remainder is 56 therefore the first polynomial is not a factor of the second polynomial.

(iii) $4x^2-5, 4x^4+7x^2+15$

$$
\begin{array}{r|l}
 & x^2 + 0x + 3 \\
\hline
4x^2-5 & 4x^4 + 0x^3 + 7x^2 + 0x + 15 \\
 & \underline{{}_{-}4x^4 \qquad {}_{+}5x^2} \\
 & \quad 0x^3 + 12x^2 + 0x + 15 \\
 & \quad \underline{{}_{-}0x^3 \qquad {}_{+}0x} \\
 & \qquad 12x^2 + 15 \\
 & \qquad \underline{{}_{-}12x^2 \ {}_{+}15} \\
 & \qquad\qquad 30
\end{array}
$$

Quotient: x^2+3
Remainder: 30
Since remainder is 30 therefore the first polynomial is not a factor of the second polynomial.

(iv) $4-z, 3z^2-13z+4$

$$\begin{array}{r|l} & -3z + 1 \\ \hline -z+4 & 3z^2 - 13z + 4 \\ & \underline{{}_{-}3z^2 \; {}_{+}12z} \\ & -z + 4 \\ & \underline{{}_{+}z \; {}_{-}4} \\ & 0 \end{array}$$

Quotient: $-3z + 1$
Remainder: 0
Since remainder is 0 therefore the first polynomial is a factor of the second polynomial.

(v) $2a - 3, 10a^2 - 9a - 5$

$$\begin{array}{r|l} & 5a + 3 \\ \hline 2a-3 & 10a^2 - 9a - 5 \\ & \underline{{}_{-}10a^2 \; {}_{+}15a} \\ & 6a - 5 \\ & \underline{{}_{-}6a \; {}_{+}9} \\ & 4 \end{array}$$

Quotient: $5a + 3$
Remainder: 4
Since remainder is 4 therefore the first polynomial is not a factor of the second polynomial.

(vi) $4y + 1, 8y^2 - 2y + 1$

$$\begin{array}{r|l} & 2y - 1 \\ \hline 4y+1 & 8y^2 - 2y + 1 \\ & \underline{{}_{-}8y^2 + {}_{-}2y} \\ & -4y + 1 \\ & \underline{{}_{+}4y \; {}_{+}1} \\ & 2 \end{array}$$

Quotient: $2y - 1$
Remainder: 2
Since remainder is 2 therefore the first polynomial is not a factor of the second polynomial.

Exercise 8.6

Q1. Divide:

$x^2 - 5x + 6$ by $x - 3$

Solution:

$$\begin{array}{r|l} & x \quad - \; 2 \\ \hline x - 3 & x^2 - 5x + 6 \\ & {}_{-}x^2 - {}_{+}3x \\ \hline & \quad - 2x + 6 \\ & \quad - {}_{+}2x + {}_{-}6 \\ \hline & \qquad\qquad 0 \end{array}$$

Quotient: $x - 2$

Remainder: 0

Q2. Divide:

$ax^2 - ay^2$ by $ax + ay$

Solution:

According to the question,

$$\frac{ax^2 - ay^2}{ax + ay}$$

$$= \frac{a(x - y)(x + y)}{a(x + y)}$$

Now cancel the common factor $a(x + y)$, we get:

$$= (x - y)$$

Therefore,

Quotient: $x - y$

Remainder: 0

Q3. Divide:

$x^4 - y^4$ by $x^2 - y^2$

Solution:

$$
\begin{array}{r|l}
 & x^2 + y^2 \\
\hline
x^2 - y^2 & x^4 - y^4 \\
 & x^4 - x^2y^2 \\
\hline
 & x^2y^2 - y^4 \\
 & x^2y^2 - y^4 \\
\hline
 & 0
\end{array}
$$

Therefore,
Quotient: $x^2 + y^2$
Remainder: 0

Q4. Divide:
$acx^2 + (bc + ad)x + bd$ by $(ax + b)$

Solution:

According to the question,

$$\frac{acx^2 + (bc + ad)x + bd}{ax + b}$$

$$= \frac{acx^2 + adx + bcx + bd}{ax + b}$$

$$= \frac{ax(cx + d) + b(cx + d)}{ax + b}$$

$$= \frac{(cx + d)(ax + b)}{ax + b}$$

$$= cx + d$$

Therefore,
Quotient: $cx + d$
Remainder: 0

Q5. Divide:
$(a^2 + 2ab + b^2) - (a^2 + 2ac + c^2)$ by $2a + b + c$

Solution:

Given: Dividend = $(a^2 + 2ab + b^2) - (a^2 + 2ac + c^2)$
Divisor = $2a + b + c$
Let's simplify the dividend.
$(a^2 + 2ab + b^2) - (a^2 + 2ac + c^2)$

$= a^2 + 2ab + b^2 - a^2 - 2ac - c^2$

$= 2ab + b^2 - 2ac - c^2$

$= 2a(b - c) + (b - c)(b + c)$

$= (b - c)(2a + b + c)$

Therefore,

$$\frac{(a^2 + 2ab + b^2) - (a^2 + 2ac + c^2)}{2a + b + c}$$

$$= \frac{(b - c)(2a + b + c)}{2a + b + c}$$

$= (b - c)$

Hence,

Quotient: $b - c$

Remainder: 0

Q6. Divide:

$\frac{1}{4}x^2 - \frac{1}{2}x - 12$ by $\frac{1}{2}x - 4$

Solution:

Given:

Dividend = $\frac{1}{4}x^2 - \frac{1}{2}x - 12$

Divisor = $\frac{1}{2}x - 4$

Let's simplify the dividend and divisor.

Dividend = $\frac{1}{4}x^2 - \frac{1}{2}x - 12 = \frac{x^2-2x-48}{4} = \frac{(x-8)(x+6)}{4}$ ...(i)

Divisor = $\frac{1}{2}x - 4 = \frac{x-8}{2}$ ...(ii)

Therefore,

$$\frac{\frac{1}{4}x^2 - \frac{1}{2}x - 12}{\frac{1}{2}x - 4}$$

$$= \frac{\frac{(x-8)(x+6)}{4}}{\frac{x-8}{2}} \quad \text{[From (i) and (ii)]}$$

$$= \frac{(x - 8)(x + 6)}{4} \times \frac{2}{(x - 8)}$$

$$= \frac{(x + 6)}{2}$$

Therefore,

Quotient: $\frac{(x+6)}{2}$

Remainder: 0