Solutions
Grade 09 Mathematics
Chapter 12: Heron's Formula

Exercise 12.1

Q1. Find the area of a triangle whose sides are respectively 150 cm, 120 cm, and 200 cm.

Solution:

Let a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:

$\mathrm{A}=\sqrt{s(s-a)(s-b)(s-c)}$ where $s=\frac{a+b+c}{2}$ [Heron's Formula]

$a=150, b=120, c=200$

$$s=\frac{a+b+c}{2}=\frac{150+120+200}{2}=235$$

$$A=\sqrt{235(235-150)(235-120)(235-200)}$$

$$A=\sqrt{235\times 85\times 115\times 35}=25\sqrt{47\times 17\times 23\times 7}=25\sqrt{128639}\ \mathrm{cm}^2$$

Q2. Find the area of a triangle whose sides are 9 cm, 12 cm and 15 cm.

Solution:

Let a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:

$\mathrm{A}=\sqrt{s(s-a)(s-b)(s-c)}$ where $s=\frac{a+b+c}{2}$ [Heron's Formula]

$a=9, b=12, c=15$

$$s=\frac{a+b+c}{2}=\frac{9+12+15}{2}=18$$

$$A=\sqrt{18(18-9)(18-12)(18-15)}$$

$$A=\sqrt{18\times 9\times 6\times 3}=54\ \mathrm{cm}^2$$

Q3. Find the area of a triangle two sides of which are 18 cm and 10 cm, and the perimeter is 42 cm.

Solution:

Let the third side of the triangle is x.

Since the perimeter of a triangle is given by:

$a+b+c=$ perimeter

$18+10+x=42$

$x=42-28$

$x = 14$ cm

Let a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:

$\mathrm{A} = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$ [Heron's Formula]

$a = 18, b = 10, c = 14$

$$s = \frac{a+b+c}{2} = \frac{18+10+14}{2} = 21$$

$$A = \sqrt{21(21-18)(21-10)(21-14)}$$

$$A = \sqrt{21 \times 3 \times 11 \times 7} = 21\sqrt{11}\ \mathrm{cm}^2$$

Q4. In a $\triangle$ ABC, $AB = 15$ cm, $BC = 13$ cm and $AC = 14$ cm. Find the area of $\triangle ABC$ and hence its altitude on AC.

Solution:

Let a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:

$\mathrm{A} = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$ [Heron's Formula]

$a = 15, b = 13, c = 14$

$$s = \frac{a+b+c}{2} = \frac{15+13+14}{2} = 21$$

$$A = \sqrt{21(21-15)(21-13)(21-14)}$$

$$A = \sqrt{21 \times 6 \times 8 \times 7} = 84\ \mathrm{cm}^2$$

Area of triangle $= \frac{1}{2}(\text{Base} \times \text{Altitude})$

$84 = \frac{1}{2}(14 \times \text{Altitude})$

Altitude $= 12$ cm

Q5. The perimeter of a triangular field is 540 m, and its sides are in the ratio 25: 17: 12. Find the area of a triangle.

Solution:

Sides of the triangle are in ratio: 25: 17: 12

$a = 25x, b = 17x, c = 12x$

Since the perimeter of a triangle is given by:

$a + b + c =$ perimeter

$25x + 17x + 12x = 540$

$$x = \frac{540}{54} = 10$$

Therefore sides of the triangle are:

$a = 25x = 25 \times 10 = 250,$

$b = 17x = 17 \times 10 = 170,$

$c = 12x = 12 \times 10 = 120$

When a, b and c are the sides of the triangle and s is the semiperimeter, then its area is given by:

$\mathrm{A} = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$ [Heron's Formula]

$$s = \frac{a+b+c}{2} = \frac{250+170+120}{2} = 270$$

$$A = \sqrt{270(270-250)(270-170)(270-120)}$$

$$A = \sqrt{270 \times 20 \times 100 \times 150} = 9000 \mathrm{~m}^2$$

Q6. The perimeter of a triangle is 300 m. If its sides are in the ratio 3: 5: 7. Find the area of the triangle.

Solution:

Sides of triangle are in ratio: 3:5:7

$a = 3x, b = 5x, c = 7x$

Since the perimeter of a triangle is given by:

$a + b + c =$ perimeter

$3x + 5x + 7x = 300$

$$x = \frac{300}{15} = 20$$

$x = 20$

Therefore sides of the triangle are:

$a = 3x = 3 \times 20 = 60,$

$b = 5x = 5 \times 20 = 100,$

$c = 7x = 7 \times 20 = 140$

When a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:

$\mathrm{A} = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$ [Heron's Formula]

$$s = \frac{a+b+c}{2} = \frac{60+100+140}{2} = 150$$

$$A = \sqrt{150(150-60)(150-100)(150-140)}$$

$$\mathrm{A} = \sqrt{150 \times 90 \times 50 \times 10} = \sqrt{15 \times 9 \times 5 \times 10000}$$

$$= \sqrt{15 \times 3 \times 3 \times 5 \times 10000}$$

$$= \sqrt{15 \times 3 \times 15 \times 10000} = 15 \times 100\sqrt{3} = 1500\sqrt{3} \mathrm{~m}^2$$

Q7. The perimeter of a triangular field is 240 dm. If two of its sides are 78 dm and 50 dm, find the length of the perpendicular on the side of length 50 dm from the opposite vertex.

Solution:

Let the third side of the triangle is x.

Since the perimeter of a triangle is given by:

$a + b + c =$ perimeter

$78 + 50 + x = 240$

$x = 240 - 128$

$x = 112$ dm

Let a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:

$\mathrm{A} = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$ [Heron's Formula]

$a = 78, b = 50, c = 112$

$$s = \frac{a+b+c}{2} = \frac{78+50+112}{2} = 120$$

$$A = \sqrt{120(120-78)(120-50)(120-112)}$$

$$A = \sqrt{120 \times 42 \times 70 \times 8} = 1680 \text{ dm}^2$$

Area of triangle $= \frac{1}{2}(\text{Base} \times \text{Altitude})$

$1680 = \frac{1}{2}(50 \times \text{Altitude})$

Altitude $= 67.2$ dm

Q8. A triangle has sides 35 cm, 54 cm and 61 cm long. Find its area. Also, find the smallest of its altitudes.

Solution:

Let a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:

$\mathrm{A} = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$ [Heron's Formula]

$a = 35, b = 54, c = 61$

$$s = \frac{a+b+c}{2} = \frac{35+54+61}{2} = 75$$

$$A = \sqrt{75(75-35)(75-54)(75-61)}$$

$$A = \sqrt{75 \times 40 \times 21 \times 14} = 939.15 \text{ cm}^2$$

Altitude on side 35 cm:

Area of triangle $= \frac{1}{2}(\text{Base} \times \text{Altitude})$

$939.15 = \frac{1}{2}(35 \times \text{Altitude})$

Altitude $= 53.66$ cm

Altitude on side 54 cm:

Area of triangle $= \frac{1}{2}(\text{ Base} \times \text{Altitude})$

$939.15 = \frac{1}{2}(54 \times \text{Altitude})$

$\text{Altitude} = 34.78 \text{ cm}$

Altitude on side 61 cm:

$\text{Area of triangle} = \frac{1}{2}(\text{Base} \times \text{Altitude})$

$939.15 = \frac{1}{2}(61 \times \text{Altitude})$

$\text{Altitude} = 30.79 \text{ cm}$

Therefore smallest Altitude is: 30.79 cm

Q9. The lengths of the sides of a triangle are in the ratio 3: 4: 5 and its perimeter is 144 cm. Find the area of the triangle and the height corresponding to the longest side.

Solution:

Sides of triangle are in ratio: 3:4:5.

$a = 3x, b = 4x, c = 5x$

Since the perimeter of a triangle is given by:

$a + b + c = \text{perimeter}$

$3x + 4x + 5x = 144$

$$x = \frac{144}{12} = 12$$

$x = 12$

Therefore sides of the triangle are:

$a = 3x = 3 \times 12 = 36,$

$b = 4x = 4 \times 12 = 48,$

$c = 5x = 5 \times 12 = 60$

When a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:

$\text{A} = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$ [Heron's Formula]

$$s = \frac{a+b+c}{2} = \frac{36+48+60}{2} = 72$$

$A = \sqrt{72(72-36)(72-48)(72-60)}$

$A = \sqrt{72 \times 36 \times 24 \times 12} = 864 \text{ cm}^2$

$\text{Area of triangle} = \frac{1}{2}(\text{Base} \times \text{Altitude})$

$864 = \frac{1}{2}(60 \times \text{Altitude})$

$\text{Altitude} = 28.8 \text{ cm}$

Q10. The perimeter of an isosceles triangle is 42 cm, and its base is $\frac{3}{2}$ times each of the equal sides. Find the length of each side of the triangle, area of the triangle and the height of the triangle.

Solution:

Let the equal sides of isosceles triangle is x.

Base of the triangle $= \frac{3x}{2}$

Since the perimeter of a triangle is given by:

$a + b + c =$ perimeter

$$x + x + \frac{3x}{2} = 42$$

$$(2x + 2x + 3x)\frac{1}{2} = 42$$

$x = 12$ cm

Therefore sides of triangle are: $a = x = 12, b = x = 12, c = \frac{3x}{2} = 18$

Let a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:

$A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$ [Heron's Formula]

$a = 12, b = 12, c = 18$

$$s = \frac{a+b+c}{2} = \frac{12+12+1g}{2} = 21$$

$A = \sqrt{21(21-12)(21-12)(21-18)}$

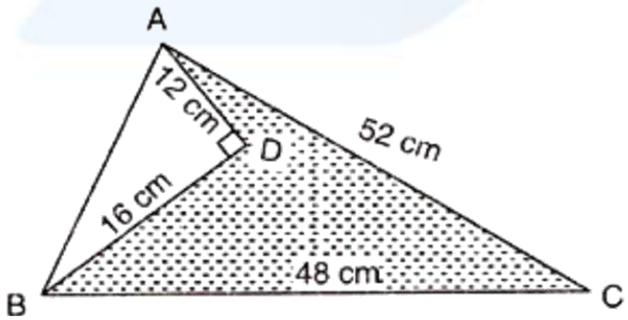
$A = \sqrt{21 \times 9 \times 9 \times 3} = 71.43 \text{ cm}^2$

Area of triangle $= \frac{1}{2}(\text{Base} \times \text{Altitude})$

$71.43 = \frac{1}{2}(18 \times \text{Altitude})$

Altitude $= 7.93$ cm

Q11. Find the area of the shaded region in the figure below.



Solution:

In right triangle ADB, side $(AB)^2 = (AD)^2 + (BD)^2$

$(AB)^2 = (12)^2 + (16)^2$

$AB = \sqrt{(12)^2 + (16)^2}$

$AB = \sqrt{144 + 256} = 20 \text{ cm}$

In right triangle ADB,

Let a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:

$\mathrm{A} = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$ [Heron's Formula]

$a = 12, b = 16, c = 20$

$$s = \frac{a+b+c}{2} = \frac{12+16+20}{2} = 24$$

$A_1 = \sqrt{24(24-12)(24-16)(24-20)}$

$A_1 = \sqrt{24 \times 12 \times 8 \times 4} = 96 \text{ cm}^2$

In right triangle ABC,

Let a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:

$\mathrm{A}_2 = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$

$a = 20, b = 48, c = 52$

$$s = \frac{a+b+c}{2} = \frac{20+48+52}{2} = 60$$

$A_2 = \sqrt{60(60-20)(60-48)(60-52)}$

$A_2 = \sqrt{60 \times 40 \times 12 \times 8} = 480 \text{ cm}^2$

Area of shaded region is $A_2 - A_1 = 480 - 96 = 384 \text{ cm}^2$

Exercise 12.2

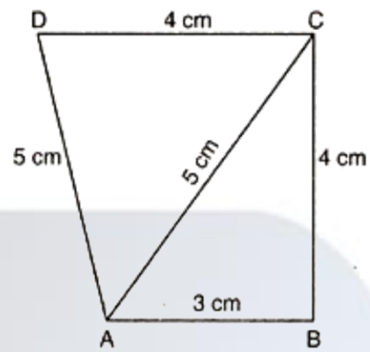
Q1. Find the area of a quadrilateral $ABCD$ is which $AB = 3$ cm, $BC = 4$ cm, $CD = 4$ cm, $DA = 5$ cm and $AC = 5$ cm.

Solution:

Let consider a quadrilateral $ABCD$.

In $\triangle$ ABC,

$AB = a = 3 \text{ cm}, BC = b = 4 \text{ cm}, AC = c = 5 \text{ cm}$



Let a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:

$\mathrm{A}=\sqrt{s(s-a)(s-b)(s-c)}$ where $s=\frac{a+b+c}{2}$ [Heron's Formula]

$s=\dfrac{a+b+c}{2}=\dfrac{3+4+5}{2}=6$

$A_1=\sqrt{6(6-3)(6-4)(6-5)}$

$A_1=\sqrt{6\times 3\times 2\times 1}=6\ \mathrm{cm}^2$

In $\triangle$ ADC;

$DA=a=5$ cm, $CD=4=4$ cm, $AC=c=5$ cm

Let a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:

$\mathrm{A}=\sqrt{s(s-a)(s-b)(s-c)}$ where $s=\frac{a+b+c}{2}$

$s=\dfrac{a+b+c}{2}=\dfrac{5+4+5}{2}=7$

$A_2=\sqrt{7(7-5)(7-4)(7-5)}$

$A_2=\sqrt{7\times 2\times 3\times 2}=9.16\ \mathrm{cm}^2$

Therefore area of quadrilateral $ABCD=A_1+A_2=6+9.16=15.16\ \mathrm{cm}^2$

Q2. The sides of a quadrangular field, taken in order are 26 m, 27 m, 7 m, are 24 m respectively. The angle contained by the last two sides is a right angle. Find its area.

Solution:

Let consider a quadrilateral $ABCD$.

In $\triangle$ ADC;

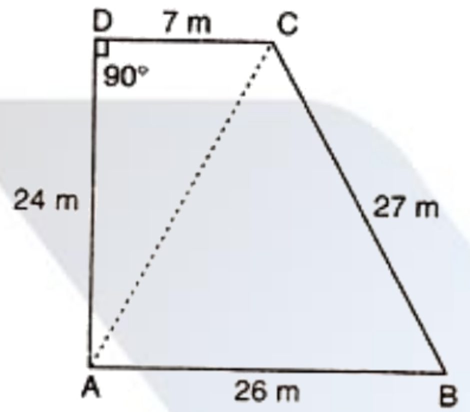
$\mathrm{AC}=\sqrt{(24)^2+7^2}=25$ cm

In $\triangle ABC$,

$AB=a=26$ cm, $BC=b=27$ cm, $AC=c=25$ cm

Let a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:

$\text{A} = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$



$s = \frac{a+b+c}{2} = \frac{26+27+25}{2} = 39$

$A_1 = \sqrt{39(39-26)(39-27)(39-25)}$

$A_1 = \sqrt{39 \times 13 \times 12 \times 14} = 291.85 \text{ cm}^2$

In $\triangle$ ADC;

$DA = a = 24 \text{ cm}, CD = b = 7 \text{ cm}, AC = c = 25 \text{ cm}$

Let a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:

$\text{A} = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$ [Heron's Formula]

$s = \frac{a+b+c}{2} = \frac{24+7+25}{2} = 28$

$A_2 = \sqrt{28(28-24)(28-7)(28-25)}$

$A_2 = \sqrt{28 \times 4 \times 21 \times 3} = 84 \text{ cm}^2$

Therefore area of quadrilateral ABCD $= \text{A}_1 + \text{A}_2 = 291.85 + 84 = 375.85 \text{ cm}^2$

Q3. The sides of a quadrilateral, taken in order are 5, 12, 14 and 15 metres respectively, and the angle contained by the first two sides is a right angle. Find its area.

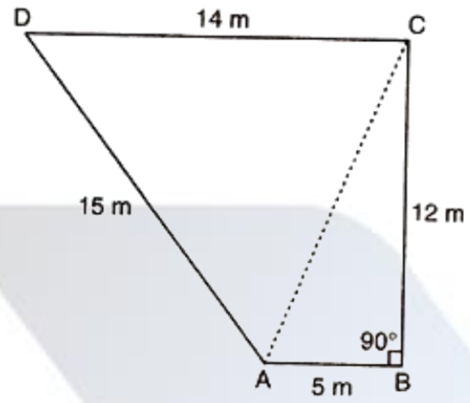
Solution:

Let consider a quadrilateral $ABCD$.

In $\triangle$ ABC;

$\text{AC} = \sqrt{5^2 + (12)^2} = 13 \text{ m}$

$AB = a = 5 \text{ m}, \text{BC} = \text{b} = 12 \text{ m}, \text{AC} = \text{c} = 13 \text{ m}$



Let a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:

$A=\sqrt{s(s-a)(s-b)(s-c)}$ where $s=\frac{a+b+c}{2}$ [Heron's Formula]

$$s=\frac{a+b+c}{2}=\frac{5+12+13}{2}=15$$

$A_1=\sqrt{15(15-5)(15-12)(15-13)}$

$A_1=\sqrt{15\times 10\times 3\times 2}=30\ \mathrm{m}^2$

In $\triangle$ ADC;

$DA=a=15\ \mathrm{m}, \mathrm{CD}=\mathrm{b}=14\ \mathrm{m}, \mathrm{AC}=\mathrm{c}=13\ \mathrm{m}$

Let a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:

$A=\sqrt{s(s-a)(s-b)(s-c)}$ where $s=\frac{a+b+c}{2}$

$$s=\frac{a+b+c}{2}=\frac{15+14+13}{2}=21$$

$A_2=\sqrt{21(21-15)(21-14)(21-13)}$

$A_2=\sqrt{21\times 6\times 7\times 8}=84\ \mathrm{m}^2$

Therefore area of quadrilateral ABCD $=\mathrm{A}_1+\mathrm{A}_2=30+84=114\ \mathrm{m}^2$

Q4. A park, in the shape of a quadrilateral $ABCD$, has $C=90^\circ, AB=9\ \mathrm{m}, BC=12\ \mathrm{m}, CD=5\ \mathrm{m}$ and $AD=8\ \mathrm{m}$. How much area does it occupy?

Solution:

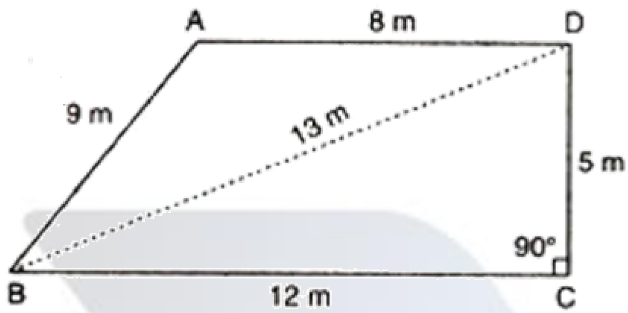
Let consider a quadrilateral ABCD.

In $\triangle$ BCD;

$BD=\sqrt{(12)^2+5^2}=13\ \mathrm{m}$

$BC=a=12\ \mathrm{m}, \mathrm{CD}=\mathrm{b}=5\ \mathrm{m}, \mathrm{BD}=\mathrm{c}=13\ \mathrm{m}$

Let a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:



$\mathrm{A}=\sqrt{s(s-a)(s-b)(s-c)}$ where $s=\frac{a+b+c}{2}$ [Heron's Formula]

$s=\frac{a+b+c}{2}=\frac{12+5+13}{2}=15$

$A_1=\sqrt{15(15-12)(15-5)(15-13)}$

$\mathrm{A}_1=\sqrt{15\times 3\times 10\times 2}=30\ \mathrm{m}^2$

In $\triangle ABD$;

$AB=a=9m, AD=b=8\ \mathrm{m}, \mathrm{BD}=\mathrm{c}=13\ \mathrm{m}$

Let a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:

$\mathrm{A}=\sqrt{s(s-a)(s-b)(s-c)}$ where $s=\frac{a+b+c}{2}$

$s=\frac{a+b+c}{2}=\frac{9+8+13}{2}=15$

$A_2=\sqrt{15(15-9)(15-8)(15-13)}$

$A_2=\sqrt{15\times 6\times 7\times 2}=35.50\ \mathrm{m}^2$

Therefore area of quadrilateral $ABCD=A_1+A_2=30+35.50=65.50\ \mathrm{m}^2$

Q5. Two parallel sides of a trapezium are 60 cm and 77 cm, and other sides are 25 cm and 26 cm. Find the area of the trapezium.

Solution:

Let $ABCD$ is a trapezium in which $AB=77$ cm, $BC=26$ cm, $CD=$ 60 cm, $DA=25$ cm.

Draw CE || AD

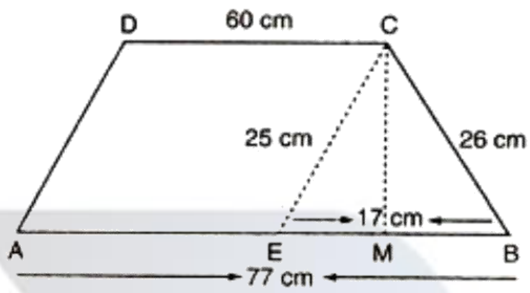
Now, ACDE is a parallelogram

$BE=AB-DC=77-60=17$ cm

In $\triangle BEC$, Let a, b and c are the sides of triangle and

And s be the semi-perimeter, then its area

$\mathrm{A}=\sqrt{s(s-a)(s-b)(s-c)}$ where $s=\frac{a+b+c}{2}$



$$s=\frac{a+b+c}{2}=\frac{25+17+26}{2}=34$$

$$A=\sqrt{34(34-25)(34-17)(34-26)}$$

$$A=\sqrt{34\times 9\times 17\times 8}=204\ \text{m}^2$$

Therefore, area of $\triangle$ BCE $=\frac{1}{2}$ (Base $\times$ Altitude)

$204\times 2=17\times$ Altitude

Altitude $=24$ cm

Area of trapezium $ABCD=\frac{1}{2}$ (Sum of parallel sides $\times$ Altitude) $=\frac{1}{2}(DC+AB)\times h$

$\Rightarrow \frac{1}{2}(60+77)\times 24=1644\ \text{cm}^2$

Therefore, area of trapezium ABCD $=1644\ \text{cm}^2$

Q6. Find the area of a rhombus whose perimeter is 80 m and one of whose diagonals is 24 m.

Solution:

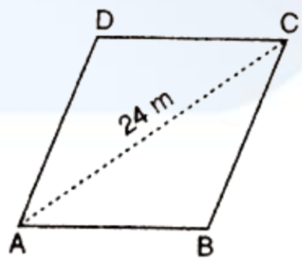
Let $ABCD$ be the rhombus of perimeter 80 m and diagonal $AC=24$ m.

We have:

$AB+BC+CD+DA=80$

$4AB=80$ [$\because AB=BC=CD=DA$ sides of Rhombus]

$AB=20$ m



In $\triangle ABC$, we have,

$AB=a=20$ cm, $BC=b=20$ cm, $AC=c=24$ cm

Let a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:

$\text{A} = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$ [Heron's Formula]

$s = \dfrac{a+b+c}{2} = \dfrac{20+20+24}{2} = 32$

$A = \sqrt{32(32-20)(32-20)(32-24)}$

$A = \sqrt{32 \times 12 \times 12 \times 8} = 192 \text{ cm}^2$

Hence, area of rhombus $ABCD = 2 \times 192 \text{ m}^2$

$= 384 \text{ m}^2$

Q7. A rhombus sheet, whose perimeter is 32 m and whose one diagonal is 10 m long, is painted on both sides at the rate of Rs. 5 per m^2. Find the cost of painting.

Solution:

Since the sides of a rhombus are equal therefore each side $= \frac{\text{Perimeter}}{4} = \frac{32}{4} = 8 \text{ m}$

Let a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:

$\text{A} = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$ [Heron's Formula]

$s = \dfrac{a+b+c}{2} = \dfrac{8+8+10}{2} = 13$

$A = \sqrt{13(13-8)(13-8)(13-10)}$

$A = \sqrt{13 \times 5 \times 5 \times 3} = 31.22 \text{ cm}^2$

Hence, area of rhombus $ABCD = 2 \times 31.22 \text{ m}^2 = 62.44 \text{ m}^2$

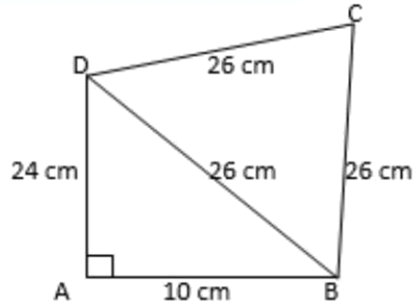
Total painting area of rhombus $= 62.44 \times 2 = 124.88 \text{ m}^2$

Cost of painting of rhombus on both sides $= 124.88 \times 5 = \text{Rs } 624.50$

Q8. Find the area of a quadrilateral $ABCD$ in which $AD = 24$ cm, $\angle BAD = 90°$ and BCD forms an equilateral triangle whose each side is equal to 26 cm. (Take $\sqrt{3} = 1.73$)

Solution:

Let $ABCD$ is a quadrilateral in which $AD = 24$ cm and $\triangle BCD$ is an equilateral.



In right angled $\triangle$ BAD applying pythagorous theorem:

$(BD)^2 = (AB)^2 + (AD)^2$

$(26)^2 = (AB)^2 + (24)^2$

$\sqrt{676 - 576} = (AB)^2$

$AB = 10$ cm

Area of right angled $\triangle$ BAD $= \frac{1}{2}$(Base $\times$ Altitude)

$\Rightarrow \frac{1}{2}(10 \times 24) = 120 \text{ cm}^2$

Now in equilateral $\triangle BCD$,

Let a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:

$\mathrm{A} = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$ [Heron's Formula]

$s = \frac{a+b+c}{2} = \frac{26+26+26}{2} = 39$

$A = \sqrt{39(39-26)(39-26)(39-26)}$

$A = \sqrt{39 \times 13 \times 13 \times 13} = 292.72 \text{ cm}^2$

Hence, area of quad $ABCD = 120 + 292.72 = 412.72 \text{ cm}^2$

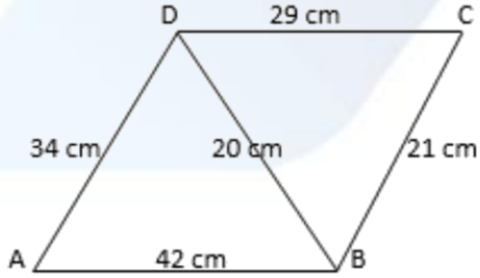
Q9. Find the area of a quadrilateral $ABCD$ in which $AB = 42$ cm, $BC = 21$ cm, $CD = 29$ cm, $DA = 34$ cm and diagonal $BD = 20$ cm.

Solution:

In $\triangle ABD$;

$AB = a = 42$ cm, $BD = b = 20$ cm, $DA = c = 34$ cm

Let a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:



$\mathrm{A} = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$

$s = \frac{a+b+c}{2} = \frac{42+20+34}{2} = 48$

$A_1 = \sqrt{48(48-42)(48-20)(48-34)}$

$A_1 = \sqrt{48 \times 6 \times 28 \times 14} = 336 \text{ cm}^2$

In $\triangle$ BCD;

$BC = a = 21 \text{ cm}, DC = b = 29 \text{ cm}, BD = c = 20 \text{ cm}$

Let a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:

$\mathrm{A} = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$

$$s = \frac{a+b+c}{2} = \frac{21+29+20}{2} = 35$$

$A_2 = \sqrt{35(35-21)(35-29)(35-20)}$

$A_2 = \sqrt{35 \times 14 \times 6 \times 15} = 210 \text{ cm}^2$

Therefore area of quadrilateral $ABCD = A_1 + A_2 = 336 + 210 = 546 \text{ cm}^2$

Q10. Find the perimeter and area of the quadrilateral $ABCD$ in which $AB = 17 \text{ cm}, AD = 9 \text{ cm}, CD = 12 \text{ cm}, ACB = 90^\circ$ and $AC = 15$ cm.

Solution:

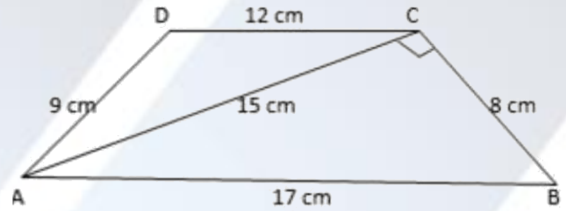
In right $\triangle$ ACB using pythagorous theorem:

$(AB)^2 = (AC)^2 + (BC)^2$

$(17)^2 = (15)^2 + (BC)^2$

$\sqrt{289 - 225} = BC$

$BC = 8$ cm



Perimeter of quad. $ABCD = AB + BC + CD + DA = 17 + 8 + 12 + 9 = 46$ cm

Area of right angled $\triangle$ ACB $= \frac{1}{2}$(Base $\times$ Altitude)

$$\Rightarrow \frac{1}{2}(15 \times 8) = 60 \text{ cm}^2$$

Now in equilateral $\triangle ACD$,

Let a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:

$\mathrm{A} = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$ [Heron's Formula]

$$s = \frac{a+b+c}{2} = \frac{9+12+15}{2} = 18$$

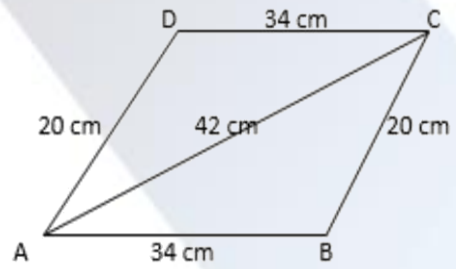
$A=\sqrt{18(18-9)(18-12)(18-15)}$

$A=\sqrt{18\times 9\times 6\times 3}=54\text{ cm}^2$

Hence, area of quad $ABCD=60+54=114\text{ cm}^2$

Q11. The adjacent sides of a parallelogram $ABCD$ measure 34 cm and 20 cm, and the diagonal AC measures 42 cm. Find the area of the parallelogram.

Solution:



In $\triangle ABC$,

$AB=a=34\text{ cm}, \text{BC}=\text{b}=20\text{ cm}, \text{AC}=\text{c}=42\text{ cm}$

Let a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:

$\text{A}=\sqrt{s(s-a)(s-b)(s-c)}$ where $s=\frac{a+b+c}{2}$

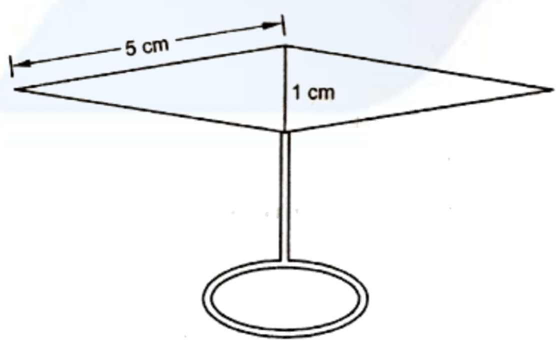
$$s=\frac{a+b+c}{2}=\frac{34+20+42}{2}=48$$

$A=\sqrt{48(48-34)(48-20)(48-42)}$

$A=\sqrt{48\times 14\times 28\times 6}=336\text{ cm}^2$

Hence, area of Parallelogram $ABCD=2\times 336=672\text{ cm}^2$

Q12. Find the area of the blades of the magnetic compass shown in figure. (Take $\sqrt{11}=3.32$)



Solution:

Let a, b and c are the sides of blade and s is the semi-perimeter, then its area is given by:

$\mathrm{A}=\sqrt{s(s-a)(s-b)(s-c)}$ where $s=\frac{a+b+c}{2}$ [Heron's Formula]

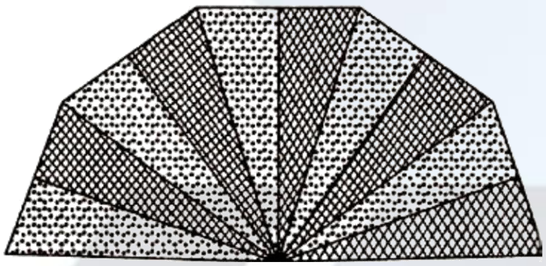
$$s=\frac{a+b+c}{2}=\frac{5+5+1}{2}=5.5$$

$$A=\sqrt{5.5(5.5-5)(5.5-5)(5.5-1)}$$

$$A=\sqrt{5.5\times0.5\times0.5\times4.5}=2.49\ \mathrm{cm}^2$$

Hence, total area of both the blades $=2\times2.49=4.98\ \mathrm{cm}^2$

Q13. A hand fan is made by stitching 10 equal size triangular strips of two different types of paper as shown in figure. The dimensions of equal strips are 25 cm, 25 cm and 14 cm. Find the area of each type of paper needed to make the hand fan.



Solution:

Let a, b and c are the sides of triangular strips and s is the semi-perimeter, then its area is given by:

$\mathrm{A}=\sqrt{s(s-a)(s-b)(s-c)}$ where $s=\frac{a+b+c}{2}$ [Heron's Formula]

$$s=\frac{a+b+c}{2}=\frac{25+25+14}{2}=32$$

$$A=\sqrt{32(32-25)(32-25)(32-14)}$$

$$A=\sqrt{32\times7\times7\times18}=168\ \mathrm{cm}^2$$

Hence, total area of 5 Nos of triangular strips of one type $=5\times168=840\ \mathrm{cm}^2$

Q14. A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 13 cm, 14 cm and 15 cm and the parallelogram stands on the base 14 cm, find the height of the parallelogram.

Solution:

Let a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:

$\mathrm{A}=\sqrt{s(s-a)(s-b)(s-c)}$ where $s=\frac{a+b+c}{2}$ [Heron's Formula]

$$s=\frac{a+b+c}{2}=\frac{13+14+15}{2}=21$$
$$A=\sqrt{21(21-13)(21-14)(21-15)}$$
$$A=\sqrt{21\times 8\times 7\times 6}=84\text{ cm}^2$$
Therefore area of $\Delta=\frac{1}{2}(\text{Base}\times\text{Altitude})$

$84\times 2=14\times\text{Altitude}$

$\text{Altitude}=12\text{ cm}$

CCE - Formative Assessment

Q1. Find the area of a triangle whose base and altitude are 5 cm and 4 cm respectively.

Solution:

Base = 5 cm, Altitude = 4 cm

Area of $\Delta=\frac{1}{2}(\text{Base}\times\text{Altitude})$

Area of $\Delta=\frac{1}{2}(5\times 4)=10\text{ cm}^2$

Q2. Find the area of a triangle whose sides are 3 cm, 4 cm, and 5 cm respectively.

Solution:

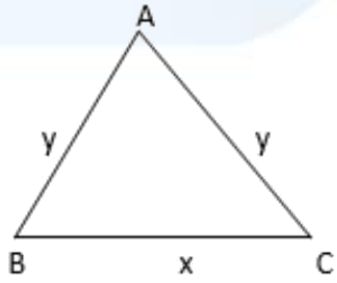
Let a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:

$\text{A}=\sqrt{s(s-a)(s-b)(s-c)}$ where $s=\frac{a+b+c}{2}$ [Heron's Formula]

$$s=\frac{a+b+c}{2}=\frac{3+4+5}{2}=6$$
$$A=\sqrt{6(6-3)(6-4)(6-5)}$$
$$A=\sqrt{6\times 3\times 2\times 1}=6\text{ cm}^2$$

Q3. Find the area of an isosceles triangle having the base x cm and one side y cm.

Solution:



In $\triangle$ ABC, AB $=y, BC=x, AC=y$

Since y, y and x are the sides of an isosceles triangle and s is the semi-perimeter, then its area is given by:

$A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$ [Heron's Formula]

$$S = \frac{x+y+y}{2} = \frac{x+2y}{2}$$

$$A = \sqrt{\frac{x+2y}{2}\left(\frac{x+2y}{2} - x\right)\left(\frac{x+2y}{2} - y\right)\left(\frac{x+2y}{2} - y\right)}$$

$$A = \sqrt{\frac{x+2y}{2}\left(\frac{2y-x}{2}\right)\left(\frac{x}{2}\right)\left(\frac{x}{2}\right)} = \frac{x}{2}\sqrt{y^2 - \frac{x^2}{4}}\ \text{cm}^2$$

Q4. Find the area of an equilateral triangle having each side 4 cm.

Solution:

Area of an equilateral triangle $= \frac{\sqrt{3}a^2}{4} = \frac{\sqrt{3}\times 4\times 4}{4} = 4\sqrt{3}\ \text{cm}^2$

Q5. Find the area of an equilateral triangle having each side x cm.

Solution:

Area of an equilateral triangle $= \frac{\sqrt{3}a^2}{4} = \frac{\sqrt{3}\times x\times x}{4} = \frac{\sqrt{3}x^2}{4}\ \text{cm}^2$

Q6. The perimeter of a triangular field is 144 m, and the ratio of the sides is 3: 4: 5. Find the area of the field.

Solution:

Sides of triangle are in ratio: 3:4:5

$a = 3x, b = 4x, c = 5x$

Since the perimeter of a triangle is given by:

$a + b + c =$ perimeter

$3x + 4x + 5x = 144$

$$x = \frac{144}{12} = 12$$

$x = 12$

Therefore sides of the triangle are:

$a = 3x = 3 \times 12 = 36,$

$b = 4x = 4 \times 12 = 48,$

$c = 5x = 5 \times 12 = 60$

When a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:

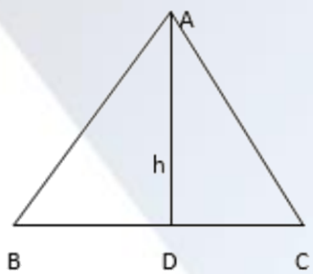
$A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$ [Heron's Formula]

$$s = \frac{a+b+c}{2} = \frac{36+48+60}{2} = 72$$
$$\text{A} = \sqrt{72(72-36)(72-48)(72-60)}$$
$$\text{A} = \sqrt{72 \times 36 \times 24 \times 12} = 864 \text{ cm}^2$$

Q7. Find the area of an equilateral triangle having altitude h cm.

Solution:

Let each side of equilateral triangle is a cm.



Using Pythagoras theorem in right △ ADB,

$$(AB)^2 = (AD)^2 + (BD)^2$$
$$(a)^2 = (h)^2 + \left(\frac{a}{2}\right)^2$$
$$a = \frac{2h}{\sqrt{3}} \text{ cm}$$

Area of an equilateral triangle $= \frac{\sqrt{3}a^2}{4} = \frac{\sqrt{3}\times 2h \times 2h}{4\times\sqrt{3}\times\sqrt{3}} = \frac{\sqrt{3}h^2}{3} = \frac{\sqrt{3}\times\sqrt{3}h^2}{3\times\sqrt{3}} = \frac{h^2}{\sqrt{3}} \text{ cm}^2$

Q8. Let Δ be the area of a triangle. Find the area of a triangle whose each side is twice the side of the given triangle.

Solution:

When each side of triangle $= a$

When a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$ [Heron's Formula]

$$s = \frac{a+b+c}{2}$$

When each side of triangle = 2a

$$s' = \frac{2a+2b+2c}{2} = \frac{2(a+b+c)}{2} = 2\text{s}$$
$$\text{A}' = \sqrt{2s(2s-2a)(2s-2b)(2s-2c)}$$
$$\text{A}' = \sqrt{2s(2s-2a)(2s-2b)(2s-2c)} = 4\sqrt{s(s-a)(s-b)(s-c)} = 4\Delta$$

Q9. If each side of a triangle is doubled, then find percentage increase in its area.

Solution:

Let the sides of triangle are a, b, c.

When a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:

$\mathrm{A}=\sqrt{s(s-a)(s-b)(s-c)}$ where $s=\frac{a+b+c}{2}$ [Heron's Formula]

$$s=\frac{a+b+c}{2}$$

When each side of triangle is doubled, then:

$$s'=\frac{2a+2b+2c}{2}=\frac{2(a+b+c)}{2}=2s$$

$$A'=\sqrt{2s(2s-2a)(2s-2b)(2s-2c)}$$

$$\mathrm{A}'=4\sqrt{s(s-a)(s-b)(s-c)}=4\mathrm{A}$$

Increase in area $=4\mathrm{A}-\mathrm{A}=3\mathrm{A}$

Percentage increase in area $=\frac{\text{Inceased area}}{\text{Original area}}\times 100=\frac{3A}{A}\times 100=300\%$

Q10. If each side of an equilateral triangle triples, then what is the percentage increase in the area of the triangle?

Solution:

When each side of triangle $=a$

Area of an equilateral triangle $=\frac{\sqrt{3}a^2}{4}=\mathrm{A}$

When each side of triangle $=3\mathrm{a}$

Area of an equilateral triangle $=\frac{\sqrt{3}(3a)^2}{4}=9\left(\frac{\sqrt{3}a^2}{4}\right)=9\mathrm{A}$

Increase in area $=9A-A=8A$

Percentage increase in area $=\frac{\text{Inceased area}}{\text{Original area}}\times 100=\frac{8A}{A}\times 100=800\%$

Q11. The sides of a triangle are 16 cm, 30 cm, 34 cm. Its area is

A. 240 cm^2

B. $225\sqrt{3}$ cm^2

C. $225\sqrt{2}$ cm^2

D. 450 cm^2

Solution:

Let a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:

$\mathrm{A}=\sqrt{s(s-a)(s-b)(s-c)}$ where $s=\frac{a+b+c}{2}$ [Heron's Formula]

$a=16, b=30, c=34$

$$s=\frac{a+b+c}{2}=\frac{16+30+34}{2}=40$$
$$A=\sqrt{40(40-16)(40-30)(40-34)}$$
$$A=\sqrt{40\times 24\times 10\times 6}=240\text{ cm}^2$$

Q12. The base of an isosceles right triangle is 30 cm. Its area is

A. 225 cm^2

B. $225\sqrt{3}\text{ cm}^2$

C. $225\sqrt{2}\text{ cm}^2$

D. 450 cm^2

Solution:

Base $=30$ cm, Altitude $=30$ cm

Area of $\Delta=\frac{1}{2}(\text{Base}\times\text{Altitude})$

Area of $\Delta=\frac{1}{2}(30\times 30)=450\text{ cm}^2$

Q13. The sides of a triangle are 7 cm, 9 cm and 14 cm. Its area is

A. $12\sqrt{5}\text{ cm}^2$

B. $12\sqrt{3}\text{ cm}^2$

C. $24\sqrt{5}\text{ cm}^2$

D. 63 cm^2

Solution:

Let a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:

$\text{A}=\sqrt{s(s-a)(s-b)(s-c)}$ where $s=\frac{a+b+c}{2}$ [Heron's Formula]

$a=7$ cm, $b=9$ cm, $c=14$ cm

$$S=\frac{a+b+c}{2}=\frac{7+9+14}{2}=15$$
$$A=\sqrt{15(15-7)(15-9)(15-14)}$$
$$A=\sqrt{15\times 8\times 6\times 1}=12\sqrt{5}\text{ cm}^2$$

Q14. The sides of a triangular field are 325 m, 300 m and 125 m. Its area is

A. 18750 m^2

B. 37500 m^2

C. 97500 m^2

D. 48750 m^2

Solution:

Let a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:

$\mathrm{A}=\sqrt{s(s-a)(s-b)(s-c)}$ where $s=\frac{a+b+c}{2}$ [Heron's Formula]

$a=325 \mathrm{~m}, b=300 \mathrm{~m}, c=125 \mathrm{~m}$

$$s=\frac{a+b+c}{2}=\frac{325+300+125}{2}=375$$

$$A=\sqrt{375(375-325)(375-300)(375-125)}$$

$$A=\sqrt{375 \times 50 \times 75 \times 250}=18750 \mathrm{~cm}^{2}$$

Q15. The sides of the triangle are 50 cm, 78 cm and 112 cm. The smallest altitude is

A. 20 cm

B. 30 cm

C. 40 cm

D. 50 cm

Solution:

Let a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:

$\mathrm{A}=\sqrt{s(s-a)(s-b)(s-c)}$ where $s=\frac{a+b+c}{2}$ [Heron's Formula]

$a=50 \mathrm{~cm}, b=78 \mathrm{~cm}, c=112 \mathrm{~cm}$

$$s=\frac{a+b+c}{2}=\frac{50+78+112}{2}=120$$

$$A=\sqrt{120(120-50)(120-78)(120-112)}$$

$$A=\sqrt{120 \times 70 \times 42 \times 8}=1680 \mathrm{~cm}^{2}$$

Area of triangle $=\frac{1}{2}(\text{Base} \times \text{Altitude})$

$1680=\frac{1}{2}(112 \times \text{Altitude})$

Altitude $=30 \mathrm{~cm}$

Q16. The sides of a triangle are 11 m, 60 m and 61 m. The altitude to the smallest side is

A. 11 m

B. 66 m

C. 50 m

D. 60 m

Solution:

Let a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:

$\text{A} = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$ [Heron's Formula]

$a = 11 \text{ m}, b = 60 \text{ m}, c = 61 \text{ m}$

$s = \frac{a+b+c}{2} = \frac{11+60+61}{2} = 66$

$A = \sqrt{66(66-11)(66-60)(66-61)}$

$A = \sqrt{66 \times 55 \times 6 \times 5} = 330 \text{ m}^2$

Area of triangle $= \frac{1}{2}(\text{Base} \times \text{Altitude})$

$330 = \frac{1}{2}(11 \times \text{Altitude})$

Altitude $= 60$ m

Q17. The sides of a triangle are 11 cm, 15 cm and 16 cm. The altitude to the largest side is

A. $30\sqrt{7}$ cm

B. $\frac{15\sqrt{7}}{2}$ cm

C. $\frac{15\sqrt{7}}{4}$ cm

D. 30 cm

Solution:

Let a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:

$\text{A} = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$ [Heron's Formula]

$a = 11 \text{ cm}, b = 15 \text{ cm}, c = 16 \text{ cm}$

$s = \frac{a+b+c}{2} = \frac{11+15+16}{2} = 21$

$A = \sqrt{21(21-11)(21-15)(21-16)}$

$A = \sqrt{21 \times 10 \times 6 \times 5} = \sqrt{3 \times 7 \times 2 \times 5 \times 2 \times 3 \times 5} = 30\sqrt{7} \text{ cm}^2$

Area of triangle $= \frac{1}{2}(\text{Base} \times \text{Altitude})$

$30\sqrt{7} = \frac{1}{2}(16 \times \text{Altitude})$

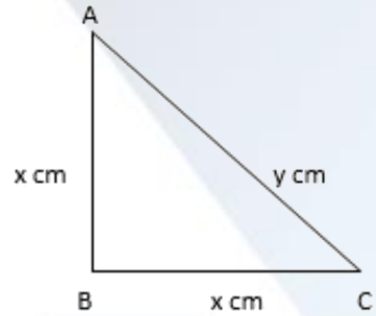
Altitude $= \frac{15\sqrt{7}}{4}$ cm

Q18. If the area of an isosceles right triangle is 8 cm^2, what is the perimeter of the triangle?

A. $8 + \sqrt{2}$ cm

B. $8 + 4\sqrt{2}$ cm

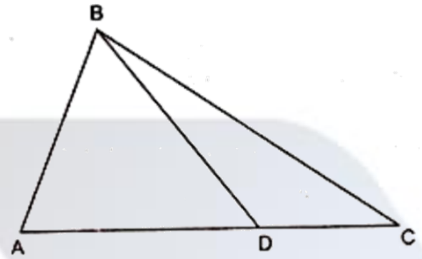
C. $4 + 8\sqrt{2}$ cm
D. $12\sqrt{2}$ cm
Solution:
In right isosceles △ ABC,
Area of triangle $= \frac{1}{2}(\text{Base} \times \text{Altitude})$
$8 = \frac{1}{2}(x \times x) = \frac{1}{2}x^2$
$x = \sqrt{16} = 4$ cm
$(AC)^2 = (AB)^2 + (BC)^2$
$y^2 = x^2 + x^2$



$y^2 = 4^2 + 4^2$
$y = \sqrt{4^2 + 4^2} = \sqrt{32} = 4\sqrt{2}$ cm
Therefore, perimeter of $\triangle ABC = AB + BC + CA = 4 + 4 + 4\sqrt{2} = 8 + 4\sqrt{2}$ cm

Q19. The length of the sides of $\triangle ABC$ are consecutive intergers. It $\triangle ABC$ has the same perimeter as an equilateral triangle, triangle with a side of length 9 cm, what is the length of the shortest side of $\triangle ABC$?
A. 4 cm
B. 6 cm
C. 8 cm
D. 10 cm
Solution:
Perimeter of an equilateral triangle with side 9 cm $= 9 \times 3 = 27$ cm
Let the sides of △ ABC are: AB $= x, BC = x + 1, AC = x + 2$ (Since the sides of △ ABC are consecutive intergers)
Perimeter of $\triangle ABC$ = Perimeter of equilateral triangle
$x + x + 1 + x + 2 = 27$
$3x = 27 - 3$
$x = 8$ cm

Q20. In the below figure, the ratio of AD to DC is 3:2. If the area of $\triangle ABC$ is 40 cm^2, what is the area of $\triangle BDC$?



A. 16 cm^2
B. 24 cm^2
C. 30 cm^2
D. 36 cm^2

Solution:

In $\triangle ABC$,

$AD : DC = 3 : 2$

Area of $\triangle$ ABD $= \frac{1}{2}(\text{Base} \times \text{Height}) = \frac{1}{2}(\text{AD} \times \text{Height})$

Area of $\triangle$ BDC $= \frac{1}{2}(DC \times \text{Height}) = \frac{1}{2}(DC \times \text{Height})$

$$\frac{\text{area of } \triangle ABD}{\text{area of } \triangle BDC} = \frac{\frac{1}{2}(AD \times \text{ Height})}{\frac{1}{2}(DC \times H\text{ eight})} = \frac{AD}{DC} = \frac{3}{2} = x$$

Area of $\triangle ABC$ = area of $\triangle ABD$ + area of $\triangle BDC = 40\text{ cm}^2$

$3x + 2x = 40$

$5x = 40$

$x = 8$

Therefore, area of $\triangle$ ABD $= 3x = 3 \times 8 = 24\text{ cm}^2$

Therefore area of $\triangle$ BDC $= 2x = 2 \times 8 = 16\text{ cm}^2$

Q21. The base and hypotenuse of a right triangle are respectively 5 cm and 13 cm long. Its area is

A. 25 cm^2
B. 28 cm^2
C. 30 cm^2
D. 40 cm^2

Solution:

Using Pythagorous theorem:

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Altitude})^2$

$(13)^2 = 5^2 + (\text{Altitude})^2$

$(\text{Altitude})^2 = 169 - 25 = 144$

Altitude $= 12$ cm

Area of triangle $= \frac{1}{2}(5 \times 12) = 30\ \text{cm}^2$

Q22. If the length of a median of an equilateral triangle is x cm, then its area is

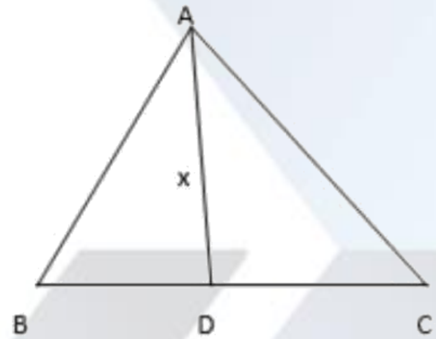
A. x^2

B. $\frac{\sqrt{3}}{2}x^2$

C. $\frac{x^2}{\sqrt{3}}$

D. $\frac{x^2}{2}$

Solution:



Median of an equilateral $= x$ cm

In an equilateral triangle median is an altitude.

Let the side of triangle be a cm.

$(AB)^2 = (AD)^2 + (BD)^2$

$(a)^2 = (x)^2 + \left(\frac{a}{2}\right)^2$

$(a)^2 - \left(\frac{a}{2}\right)^2 = (x)^2$

$\frac{3}{2}a^2 = x^2$

$a = \frac{2x}{\sqrt{3}}$ cm

Area of an equilateral triangle $= \frac{\sqrt{3}a^2}{4} = \frac{\sqrt{3} \times 2x \times 2x}{4 \times \sqrt{3} \times \sqrt{3}} = \frac{\sqrt{3}x^2}{3} = \frac{\sqrt{3} \times \sqrt{3}x^2}{3 \times \sqrt{3}} = \frac{x^2}{\sqrt{3}}$ cm^2

Q23. The length of each side of an equilateral triangle of area $4\sqrt{3}$ cm^2, is

A. 4 cm

B. $\frac{4}{\sqrt{3}}$ cm

C. $\frac{\sqrt{3}}{4}$ cm

D. 3 cm

Solution:

Area of an equilateral triangle $= 4\sqrt{3}\ \text{cm}^2$

Let side of equilateral triangle $= a$ cm

Area of an equilateral triangle $= \frac{\sqrt{3}a^2}{4} = 4\sqrt{3}$

$$\Rightarrow a^2 = \frac{4 \times 4\sqrt{3}}{\sqrt{3}}$$

$\Rightarrow a = 4$ cm

Q24. If every side of a triangle is doubled, then increase in the area of the triangle is

A. $100\sqrt{2}\%$

B. 200%

C. 300%

D. 400%

Solution:

Let the sides of triangle are a, b, c.

When a, b and c are the sides of triangle and s is the semi-perimeter, then its area is given by:

$\text{A} = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$ [Heron's Formula]

$$s = \frac{a+b+c}{2}$$

When each side of triangle is doubled, then:

$$s' = \frac{2a+2b+2c}{2} = \frac{2(a+b+c)}{2} = 2s$$

$$A' = \sqrt{2s(2s-2a)(2s-2b)(2s-2c)}$$

$$\text{A}' = 4\sqrt{s(s-a)(s-b)(s-c)} = 4\text{A}$$

Increase in area $= 4\text{ A} - \text{A} = 3\text{A}$

Percentage increase in area $= \frac{\text{Inceased area}}{\text{Original area}} \times 100 = \frac{3A}{A} \times 100 = 300\%$

Q25. A square and an equilateral triangle have equal perimeters. If the diagonal of the square is $12\sqrt{2}$ cm, then area of the triangle is

A. $24\sqrt{2}\ \text{cm}^2$

B. $24\sqrt{3}\ \text{cm}^2$

C. $48\sqrt{3}\ \text{cm}^2$

D. $64\sqrt{3}\ \text{cm}^2$

Solution:

Let each side of square $= y$ cm and each side of an equilateral triangle $= x$ cm

Perimeter of square = perimeter of an equilateral triangle

$4y = 3x$ ...(1)

Diagonal of square $= 12\sqrt{2}$ cm

Therefore using Pythagorous theorem:

$y^2 + y^2 = (12\sqrt{2})^2$

$2y^2 = 288$

$y = 12$ cm

Therefore substituting value of y in equation (1) we get: $x = 16$ cm

Area of an equilateral triangle $= \frac{\sqrt{3}a^2}{4} = \frac{\sqrt{3}\times 16\times 16}{4} = 64\sqrt{3}\ \text{cm}^2$