Solutions
Grade 9 Maths
Chapter 20: Surface Area and Volume of a Right Circular Cone

Exercise 20.1

Q1. Find the curved surface area of a cone, if its slant height is 60 cm and the radius of its base is 21 cm.

Solution:

Given,

Slant height of cone $(r) = 60$ cm

Radius of base of cone $(l) = 21$ cm

Curved surface area of cone $= \pi r l = \frac{22}{7} \times 21 \times 60 = 3960 \text{ cm}^2$

Q2. The radius of a cone is 5 cm, and vertical height is 12 cm. Find the area of the curved surface.

Solution:

Radius of cone $(r) = 5$ cm

Vertical height $(h) = 12$ cm

Slant height of cone $(l) = \sqrt{(r^2 + h^2)}$

$= \sqrt{(5^2 + 12^2)}$

$= \sqrt{25}$

$= \sqrt{169} = 13$ cm

Curved surface area of cone $= \pi r l = \frac{22}{7} \times 5 \times 13 = 204.28 \text{ cm}^2$

Q3. The radius of a cone is 7 cm and area of curved surface is 176 cm^2. Find the slant height.

Solution:

Given,

Radius of base $(r) = 7$ cm

Area of curved surface $= \pi r l = 176$ cm

$\therefore \dfrac{22}{7} \times 7 \times l = 176 = l = 8$ cm

Q4. The height of a cone is 21 cm. Find the area of the base if the slant height is 28 cm.

Solution:

Given,

Height of cone $(h) = 21$ cm

Slant height $(l) = 28$ cm
$\therefore P = r^2 + h^2 = r = \sqrt{l^2 - h^2} = \sqrt{28^2 - 21^2} = 7\sqrt{7}$ cm
$\therefore$ Area of the base $= \pi r^2 = \frac{22}{7} \times (7\sqrt{7})^2 \text{ cm}^2 = 1078 \text{ cm}^2$

Q5. Find the total surface area of a right circular cone with radius 6 cm and height 8 cm .

Solution:

We have,
Radius of right circular cone $(r) = 6$
Height of cone $(h) = 8$ cm
$\therefore$ slant height $(l) = \sqrt{r^2 + h^2}$
$= \sqrt{36 + 64} = 10$ cm
Total surface area of cone $= \pi r^2 + \pi r l = \pi r(r + l)$
$= \frac{22}{7} \times 6 \times 16 \text{ cm}^2 = 301.71 \text{ cm}^2$

Q6. Find the curved surface area of a cone with base radius 5.25 cm and slant height 10 cm.

Solution:

Given,
Base radius of cone $(r) = 5.25$ cm
Slant height of cone $(l) = 10$ cm
$\therefore$ Curved surface area $= \pi r l$
$= \frac{22}{7} \times 5.25 \times 10 \text{ cm}^2 = 165 \text{ cm}^2$

Q7. Find the total surface area of a cone, if its slant height is 21 m and diameter of its base is 24 m.

Solution:

We have,
Slant height of cone$(l) = 21$ m
Radius of cone $(r) = 12$ m
Total surface area $= \pi r^2 + \pi r l = \pi r(r + l)$
$= \frac{22}{7} \times 12(12 + 21) \text{m}^2$
$= \frac{22}{7} \times 12 \times 33 \text{ cm}^2 = 1244.57 \text{ m}^2$

Q8. The area of the curved surface of a cone is $60\pi \text{ cm}^2$. If the slant height of the cone be 8 cm, find the radius of the base.

Solution:

We have,
Curved surface area of cone $= 60\pi \text{ cm}^2$

Slant height $l = 8$ cm
So,$\pi rl = 60\pi$
$rl = 60$
$= r = \frac{60}{8} = 7.5$ cm

Q9. The curved surface area of a cone is 4070 cm^2 and its diameter is 70 cm. What is its slant height? (Use $\pi = \frac{22}{7}$).

Solution:

We have,
Area of curved surface $= \pi rl = 4070\ cm^2$
Radius of base $r = \frac{70}{2} = 35$ cm
$\Rightarrow \frac{22}{7} \times 35 \times l = 4070$
$\Rightarrow l = 37$ cm

Q10. The radius and slant height of a cone are in the ratio of 4: 7. If its curved surface area is 792 cm^2, find its radius. (Use $\pi = \frac{22}{7}$)

Solution:

We have,
Let radius of cone $r = 4x$,
Let slant height $l = 7x$
Area of curved surface $= \pi rl = 792\ cm^2$
Now, $\pi rl = 792$
$= \frac{22}{7} \times 4x \times 7x = 792 = x^2 = 9 = \text{x} = 3$ cm
$\therefore r = 4 \times 3 = 12$ cm
$l = 7 \times 3 = 21$ cm

Q11. A Joker's cap is in the form of a right circular cone of base radius 7 cm and height 24 cm. Find the area of the sheet required to make 10 such caps.

Solution:

We have,
Based radius of conical cap $= r = 7$ cm
Vertical height $= h = 24$ cm
$\therefore$ slant height $l = \sqrt{r^2 + h^2} = \sqrt{49 + 576} = 25$ cm
Required area of sheet =
$10 \times \pi rl = 10 \times \frac{22}{7} \times 7 \times 25\ cm^2 = 5500\ cm^2$

Q12. Find the ratio of the curved surface areas of two cones if their diameters of the bases are equal and slant heights are in the ratio 4:3.

Solution:

We have,

$r_1 = r_2$

let , $l_1 = 4x, l_2 = 3x$

$\frac{S_1}{S_2} = \frac{\pi r_1 l_1}{\pi r_2 l_2} = \frac{l_1}{l_2} = \frac{4}{3}$

Q13. There are two cones. The curved surface area of one is twice that of the other. The slant height of the later is twice that of the former. Find the ratio of their radii.

Solution:

We have,

$\mathrm{A}_1 = 2\mathrm{A}_2$

$\pi r_1 l_1 = 2(\pi r_2 h_2)$

$h_2 = 2h_1$

$r_1 l_1 = 2r_2 \times 2l_1$

$r_1 = 4r_2$

$\frac{r_1}{r_2} = \frac{4}{1}$

Q14. The diameters of two cones are equal. If their slant heights are in the ratio 5:4, find the ratio of their curved surfaces.

Solution:

We have,

$r_1 = r_2$

$l_1 : l_2 = 5 : 4$

$\frac{S_1}{S_2} = \frac{\pi r_1 l_1}{\pi r_2 l_2} = \frac{\pi r_2 \times \frac{5}{4} l_2}{\pi r_2 l_2} = \frac{5}{4}$

Q15. Curved surface area of a cone is 308 cm^2 and its slant height is 14 cm . Find the radius of the base and total surface area of the cone.

Solution:

We have,

Curved surface area of cone $\pi rl = 308\ \mathrm{cm}^2$

Slant height $l = 14$ cm

$\frac{22}{7} \times r \times 14 = 308 = r = 7\ \mathrm{cm}$

Total surface area $= \pi rl + \pi r^2 = 308 + \frac{22}{7} \times 7^2$

$= 462\ \mathrm{cm}^2$

Q16. The slant height and base diameter of a conical tomb are 25 m and 14 m respectively. Find the cost of whitewashing its curved surface at the rate of Rs. 210 per 100 m^2.

Solution:

We have,

Radius of conical tomb $= \frac{diameter}{2} = \frac{14}{2} = 7$ m

Slant height $= l = 25$ m

Curved surface area $= \pi r l$

Cost of white washing $=$ (curved surface area $\times$ rate of white washing per m^2)

$= \left\{(\pi r l) \times \frac{210}{100}\right\}$

$= \left\{\left(\frac{22}{7} \times 7 \times 25\right) \times \frac{210}{100}\right\} =$ Rs. 1155

Q17. A conical tent is 10 m high and the radius of its base is 24 m . Find the slant height of the tent. If the cost of 1 m^2 canvas is Rs. 70, find the cost of the canvas required to make the tent.

Solution:

We have,

Radius of base of tent $= r = 24$ m

Vertical height $= h = 10$ m

$\therefore$ slant height $= l = \sqrt{r^2 + h^2} = \sqrt{576 + 100} = 26$ m

$\therefore$ Cost of canvas required $=$ Rs. $\left\{\left(\frac{22}{7} \times 24 \times 26\right) \times 70\right\} =$ Rs. 137280

Q18. A tent is in the form of a right circular cylinder surmounted by a cone. The diameter of cylinder is 24 m . The height of the cylindrical portion is 11 m while the vertex of the cone is 16 m above the ground. Find the area of the canvas required for the tent.

Solution:

For cylindrical part, we have

Diameter $= 24$ m so, radius $r = \frac{24}{2} = 12$ m,

Height $= h = 11$ m

For conical part, we have

Height of the cone $= (16 - 11)$m $= 5$ m

$\therefore$ Slant height $= \sqrt{12^2 + 5^2}$ m $= 13$ m

Hence, Area of the canvas required

$=$ Curved surface area of cone + Curved surface area of cylinder

$= \pi r l + 2\pi r h = \pi r(l + 2h) = \frac{22}{7} \times 12 \times (13 + 22)\text{m}^2 = 1320\ \text{m}^2$

Q19. A circus tent is cylindrical to a height of 3 metres and conical above it. If its diameter is 105 m and the slant height of the conical portion is 53 m, calculate the length of the canvas 5 m wide to make the required tent.

Solution:

For cylindrical part, we have

Height $(h)=3$ m

Radius $(r)=\frac{105}{2}$ m

$\therefore$ Total curved surface area $=2\pi rh+\pi rl$

$=\pi r(2h+l)=\frac{22}{7}\times\frac{105}{2}\times(6+53)\mathrm{m}^2=11\times 15\times 59\ \mathrm{m}^2$

Hence, length of 5 m wide canvas $=\frac{11\times 15\times 59}{5}$ m $=1947$ m

Q20. The circumference of the base of a 10 m height conical tent is 44 metres. Calculate the length of canvas used in making the tent if width of canvas is 2 m. (Use $\pi=\frac{22}{7}$).

Solution:

We have,

Circumference of circular base of cone $=2\pi r=44$ m

Height $(h)=10$ m

$=2\times\frac{22}{7}\times r=44=r=7$ m

Slant height $(l)=\sqrt{(10^2+7^2)}=\sqrt{149}=12.20$ m

Curved surface area of tent $=\pi rl=\frac{22}{7}\times 7\times 12.20=268.4\ \mathrm{m}^2$

Hence, length of 2 m wide canvas needed $=\frac{268.4}{2}=134.2$ m

Q21. What length of tarpaulin 3 m wide will be required to make a conical tent of height 8 m and base radius 6 m? Assume that the extra length of material will be required for stitching margins and wastage in cutting is approximately 20 cm. (Use $\pi=3.14$).

Solution:

we have,

Height of tent $=8$ m

Base radius $=6$ m

Slant height $(l)=\sqrt{r^2+h^2}=\sqrt{36+64}=10$ m

Curved surface area of the cone $=\pi rl=3.14\times 6\times 10\ \mathrm{m}^2$

$\therefore$ Length of 3 m wide tarpaulin required

$=\frac{3.14\times 6\times 10}{3}=62.8$ m

Extra length required for stitching and cutting wastage $=0.2$ m

Total length required $=62.8+0.2=63$ m

Q22. A bus stop is barricaded from the remaining part of the road, by using 50 hollow cone made of recycled cardboard. Each cone has a base diameter of 40 cm and height 1 m . If the outer side of each of the cones is to be painted and the cost of painting is Rs. 12 per m^2. What will be the cost of painting all these cones. (Use $\pi=3.14$ and $\sqrt{1.04}=1.02$).

Solution:

We have,

Radius of base = 20 cm = 0.2 m

Height of cone = 1 m

$\therefore$ slant height $l = \sqrt{r^2 + h^2} = \sqrt{0.04 + 1}$ m $= \sqrt{1.04}$ m

= 1.02 m

Curved surface area of a cone $= \pi r l = 3.14 \times 0.2 \times 1.02$ m^2

Cost of painting = Rs. $[(3.14 \times 0.2 \times 1.02 \times 12) \times 50]$ = Rs. 384.33

Q23. A cylinder and a cone have equal radii of their bases and equal heights. If their curved surface areas are in the ratio 8: 5, Show that the radius of each is to the height of each as 3: 4.

Solution:

For cylinder, we have

Base radius $= r$

Height $= h$

$\therefore$ Curved surface area $S_1 = 2\pi r h$

For cone, we have

Slant height $l = \sqrt{r^2 + h^2}$

$S_2 = \pi r l = \pi r \sqrt{r^2 + h^2}$

We have,

$\frac{S_1}{S_2} = \frac{8}{5} = \frac{2\pi r h}{\pi r \sqrt{r^2 + h^2}}$

$= \frac{2h}{\sqrt{r^2 + h^2}} = \frac{8}{5}$

(squaring both side)

$= \frac{4h^2}{r^2 + h^2} = \frac{64}{25}$

$= 100h^2 = 64r^2 + 64h^2$

$= 36h^2 = 64r^2$

(Square root both side)

$= 6h = 8r$

$= \frac{r}{h} = \frac{3}{4}$

Exercise 20.2

Q1. Find the volume of a right circular cone with :

(i) radius 6 cm , height 7 cm .

(ii) radius 3.5 cm , height 12 cm

(iii) height 21 cm and slant height 28 cm .

Solution:

(i) we have,

Radius of cone $= 6$ cm
Height of cone $= 7$ cm
So, volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 36 \times 7 = 264 \text{ cm}^3$
(ii) we have,
radius $= 3.5$ cm
height $= 12$ cm
volume of cone $= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 12 = 154 \text{ cm}^3$
(iii) we have,
height $= 21$ cm
slant height $= 28$ cm
radius $r = \sqrt{(28^2 - 21^2)} = \sqrt{49 \times 7} = 7\sqrt{7}$ cm
volume of cone $= \frac{1}{3} \times \frac{22}{7} \times 7\sqrt{7} \times 7\sqrt{7} \times 21 = 7546 \text{ cm}^3$

Q2. Find the capacity in litres of a conical vessel with :
(i) radius 7 cm , slant height 25 cm
(ii) height 12 cm , slant height 13 cm .
Solution:
(i) we have,
radius of cone $= 7$ cm
slant height $= 25$ cm
vertical height of cone $h = \sqrt{(25^2 - 7^2)} = \sqrt{576} = 24$ cm
hence capacity (volume) of cone $= \frac{1}{3} \times \frac{22}{7} \times 49 \times 24 = 1232 \text{ cm}^3$
(ii) we have,
height $= 12$ cm
slant height $= 13$ cm
radius of cone $r = \sqrt{(13^2 - 12^2)} = \sqrt{25} = 5$ cm
volume of conical vessel $= \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 12 = \frac{2200}{7} = 314.28 \text{ cm}^3$

Q3. Two cones have their heights in the ratio 1: 3 and the radii of their bases in the ratio 3: 1. Find the ratio of their volumes.
Solution:
Let the heights of the cones be h and $3h$ units and the radii of their bases be $3r$ and r respectively. Then, their volumes are
$V_1 = \frac{1}{3}\pi(3r)^2 \times h$ and $V_2 = \frac{1}{3}\pi \times r^2 \times 3h$

$$= \frac{V_1}{V_2} = \frac{3\pi r^2 h}{\pi r^2 h} = \frac{3}{1}$$

Q4. The radius and the height of a rightr circular cone are in the ratio 5:12. If its volume is 314 cubic metre, find the slant height and the radius (Use $\pi = 3.14$).
Solution:

We have,
Let radius of cone $r = 5x$
Let height $h = 12x$
$\therefore$ Volume $= 314 \text{ m}^3$
$= \frac{1}{3} \times 3.14 \times (5x)^2 \times 12x = 314$
$= x^3 = 1 = x = 1$
$\therefore r = 5$ and $h = 12$
Now, slant height $l = \sqrt{r^2 + h^2} = l = \sqrt{25 + 144} = 13 \text{ m}$

Q5. The radius and height of a right circular cone are in the ratio 5:12 and its volume is 2512 cubic cm. Find the slant height and radius of the cone. (Use $\pi = 3.14$)

Solution:

Let radius $r = 5x$
Let height $h = 12x$
We have,
Volume $= 2512 \text{ m}^3$
$= \frac{1}{3} \times 3.14 \times 25x^2 \times 12x = 2512$
$= x^3 = 8 = x = 2$
$\therefore r = 10 \text{ cm}$ and $h = 24 \text{ cm}$
Hence, slant height $l = \sqrt{r^2 + h^2} = \sqrt{100 + 576} = 26 \text{ cm}$

Q6. The ratio of volumes of two cones is 4: 5 and the ratio of the radii of their bases is 2: 3. Find the ratio of their vertical heights.

Solution:

We have,
$V_1 : V_2 = 4 : 5$
$r_1 = r_2 = 2 : 3$

$$\therefore \frac{V_1}{V_2} = \frac{4}{5} = \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{4}{5} = \frac{4h_1}{9h_2} = \frac{4}{5} = \frac{h_1}{h_2} = \frac{9}{5}$$

Q7. A cylinder and a cone have equal radii of their bases and equal heights. Show that their volumes are in the ratio 3:1.

Solution:

We have,
$r_1 = r_2$
$h_1 = h_2$

$$\therefore \frac{\text{volume of cylinder}}{\text{volume of the cone}} = \frac{\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{3}{1}$$

Q8. If the radius of the base of a cone is halved, keeping the height same, what is the ratio of the volume of the reduced cone to that of the original cone?

Solution:

Let r be the radius of the base and h be the height of the original cone. Then,

$v_1 = \frac{1}{3}\pi r^2 h$

Let V_2 be the volume of the reduces cone. Then,

$v_2 = \frac{1}{3}\pi\left(\frac{r}{2}\right)^2 \times h$

$\therefore \frac{v_1}{v_2} = \frac{4}{1} = \frac{v_1}{v_2} = \frac{4}{1}$

Hence ratio of reduce cone to original cone $= 1:4$

Q9. A heap of wheat is in the form of a cone of diameter 9 m and height 3.5 m . Find its volume. How much canvas cloth is required to just cover the heap? (Use $\pi = 3.14$)

Solution:

We have,

radius $= \frac{9}{2} = 4.5$ m

height $= 3.5$ m

$\therefore$ slant height $l = \sqrt{r^2 + h^2} = \sqrt{(4.5)^2 + (3.5)^2} = 5.70$ m

$\therefore$ Volume of the heap $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times 3.14 \times (4.5)2 \times 3.5 = 74.1825\ \text{m}^3$

Area of canvas required $= \pi r l$

$= 3.14 \times 4.5 \times 5.7\ \text{m}^2 = 80.54\ \text{m}^2$

Q10. Find the weight of a solid cone whose base is of diameter 14 cm and vertical height 51 cm, supposing the material of which it is made weighs 10 grams per cubic cm.

Solution:

We have,

Radius $r = \frac{14}{2} = 7$ cm,

Height $h = 51$ cm

$\therefore$ Weight of solid cone $= \left(\frac{1}{3} \times \frac{22}{7} \times 7^2 \times 51\right) \times \frac{10}{1000}$ kg $= 26.180$ kg

Q11. A right angled triangle of which the sides containing the right angle are 6.3 cm and 10 cm in length, is made to turn round on the longer side. Find the volume of the solid, thus generated. Also, find its curved surface.

Solution:

We have,

$r = 6.3$ cm, $h = 10$ cm

$\therefore l = \sqrt{r^2 + h^2}$ cm $= \sqrt{(6.3)^2 + 100}$ cm

$= \sqrt{139.69}$ cm $= 11.82$ cm

Now, Volume $= \frac{1}{3} \times \frac{22}{7} \times (6.3)^2 \times 10 \text{ cm}^3$
$= 415.8 \text{ cm}^3$
Curved surface area $= \frac{22}{7} \times 6.3 \times 11.82 \text{ cm}^2$
$= 234.03 \text{ cm}^2$

Q12. Find the volume of the largest right circular cone that can be fitted in a cube whose edge is 14 cm.

Solution:

We have,
Radius of the base of the cone $= \frac{14}{2} = 7 \text{ cm}$
Height of the cone $= 14 \text{ cm}$
$\therefore$ Volume of the cone $= \frac{1}{3} \times \frac{22}{7} \times 7^2 \times 14 \text{ cm}^3$
$= 718.66 \text{ cm}^3$

Q13. The volume of a right circular cone is 9856 cm 3 . If the diameter of the base is 28 cm, find :
(i) height of the cone
(ii) slant height of the cone
(iii) curved surface area of the cone.

Solution:

We have,
Volume of right circular cone $= 9856 \text{ cm}^3$,
Radius of cone $= 14 \text{ cm}$
$$\therefore \frac{1}{3} \times \frac{22}{7}(14)^2 \times h = 9856$$
$h = 48 \text{ cm}$
slant height $l = \sqrt{r^2 + h^2} = \sqrt{196 + 2304} = 50 \text{ cm}$
Curved Surface area of cone $= \pi r l = \frac{22}{7} \times 14 \times 50 \text{ cm}^2 = 220 \text{ cm}^2$

Q14. A conical pit of top diameter 3.5 m is 12 m deep. What is its capacity in kilolitres?

Solution:

We have,
Radius of conical pit $= \frac{3.5}{2} = \frac{7}{4}$ m
Depth of pit ' h ' $= 12$ m
Volume $= \frac{1}{3}\pi r^2 h$
$$= \frac{1}{3} \times \frac{22}{7} \times 1.75 \times 1.75 \times 12$$
$= 38.5 \text{ m}^3$
Since, $1 \text{ m}^3 = 1$ kiloliter
Capacity of the pit $= (38.5 \times 1)$
$= 38.5$ kilolitres

Q15. Monica has a piece of Canvas whose area is 551 m^2. She uses it to have a conical tent made, with a base radius of 7 m. Assuming that all the stitching margins and wastage incurred while cutting, amounts to approximately 1 m^2. Find the volume of the tent that can be made with it.

Solution:

We have,

Area of canvas $= 551 \text{ m}^2$

Area of canvas lost in wastage $= 1 \text{ m}^2$

$\therefore$ Area of canvas used in making tent

$= (551 - 1)\text{m}^2 = 550 \text{ m}^2$

$=$ Surface area of the cone $= 550 \text{ m}^2$

We have,

$r =$ radius of the base of the cone $= 7$ m

$\therefore$ Surface area $= 550 \text{ m}^2 = \pi r l = 550$

$= \frac{22}{7} \times 7 \times l = 550 = l = 25 \text{ m}$

Let h be the height of the cone. Then,

$l = r^2 + h^2$

$h = \sqrt{l^2 - r^2} = \sqrt{25^2 - 7^2} = 24 \text{ m}$

$\therefore$ Volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 24 \text{ m}^3 = 1232 \text{ m}^3$

CCE - Formative Assessment

Q1. The height of a cone is 15 cm If its volume is $500\pi \text{ cm}^3$, then find the radius of its base.

Solution:

We have,

Height of cone $= 15$ cm

Volume of cone $= 500\pi \text{ cm}^3$

$= \frac{1}{3}\pi r^2 h = 500\pi$

$= r^2 = 100 r = \sqrt{100}$

Hence, radius of base $r = 10$ cm,

Q2. If the volume of a right circular cone of height 9 cm is $48\pi \text{ cm}^3$, find the diameter of its base.

Solution:

We have,

Volume of cone $= 48\pi \text{ cm}^3$

Height of cone h $= 9$ cm

$= \frac{1}{3}\pi r^2 h = 48\pi$

$= r^2 = 16r = 4$ cm

Hence diameter of its base $= 2 \times$ radius $= 2 \times 4 = 8$ cm

Q3. If the height and slant height of a cone are 21 cm and 28 cm respectively. Find its volume.

Solution:

We have,

Height of cone $h = 21$ cm

Slant height $l = 28$ cm

Radius of cone $= \sqrt{(l^2 - h^2)} = \sqrt{(28^2 - 21^2)} = \sqrt{343} = 7\sqrt{7}$ cm

Volume of cone $= \frac{1}{3} \times \pi \times 7\sqrt{7} \times 7\sqrt{7} \times 21$

$= 2401\pi$ cm^3

Q4. The height of a conical vessel is 3.5 cm. If its capacity is 3.3 litres of milk. Find the diameter of its base.

Solution:

We have

Height of conical vessel $= 3.5$ cm

Volume $= 3.3$ litre $= 3300$ cm^3

$= \frac{1}{3}\pi r^2 h = 3300$

$r^2 = 3300 \times 3 \times \frac{7}{22} \times 3.5 = 900$

$r = \sqrt{900} = 30$ cm

Hence diameter of its base $= 2 \times$ radius $= 2 \times 30 = 60$ cm

Q5. If the radius and slant height of a cone are in the ratio 7:13 and its curved surface area is 286 cm^2, find its radius.

Solution:

We have

Let radius of cone $= 7x$

Let slant height $= 13x$

Curved surface area of cone $= 286$ cm^2

$= \pi r l = 286,$

$\frac{22}{7} \times 7x \times 13x = 286$

$x^2 = 1$

$x = 1$

Hence radius of cone $= 7 \times 1 = 7$ cm

Q6. Find the area of canvas required for a conical tent of height 24 m and base radius 7 m.

Solution:

We have,
Height of conical tent = 24 m
Radius of its base = 7 m
Slant height of cone $l = \sqrt{(r^2 + h^2)} = \sqrt{(24^2 + 7^2)} = \sqrt{625} = 25$ m
Area of canvas required $= \pi r l = \frac{22}{7} \times 7 \times 25 = 550\ \text{m}^2$

Q7. Find the area of metal sheet required in making a closed hollow cone of base radius 7 cm and height 24 cm.

Solution:

We have,
Base radius of cone = 7 cm
Height of cone = 24 cm
Slant height of cone $l = \sqrt{(r^2 + h^2)} = \sqrt{(24^2 + 7^2)} = \sqrt{625} = 25$ cm
Hence area of metal sheet required = total surface area of cone

$$\pi r(l + r) = \frac{22}{7} \times 7(25 + 7)$$

$= 704\ \text{cm}^2$

Q8. Find the length of cloth used in making a conical pandal of height 100 m and base radius 240 m, if the cloth is 100π m wide.

Solution:

We have,
Height of conical pandal = 100 m
Base radius of pandal = 240 m
Slant height $l = \sqrt{(r^2 + h^2)} = \sqrt{240^2 + 100^2} = \sqrt{67600} = 260$ m
Curved surface area of cloth $= \pi r l = \pi \times 260 \times 240$
Hence length of 100π m wide cloth $= \frac{curved\ surface\ area\ of\ cloth}{width\ of\ cloth}$

$$= \frac{\pi \times 260 \times 240}{100\pi}$$

$= 26 \times 24 = 624$ m

Q9. The number of surfaces of a cone has, is

A. 1
B. 2
C. 3
D. 4

Solution:

A cone has two surfaces,
(i) Curved surface
(ii) Circular base

Q10. The area of the curved surface of a cone of radius $2r$ and slant height $\frac{1}{2}$, is

A. πrl

B. $2\pi rl$

C. $\frac{1}{2}\pi rl$

D. $\pi(r+h)r$

Solution:

Radius of cone $= 2r$

Slant height $= \frac{1}{2}$

Area of curved surface $= \pi rl$

$= \frac{22}{7} \times 2r \times \frac{1}{2} = \pi rl$

Q11. The total surface area of a cone of radius $\frac{r}{2}$ and length $2l$, is

A. $2\pi r(l+r)$

B. $\pi r\left(l+\frac{r}{4}\right)$

C. $\pi r(l+r)$

D. $2\pi rl$

Solution:

Radius of cone $= \frac{r}{2}$

Length of cone $= 21$

Total surface area of cone $= \pi rl + \pi r^2$

$= \pi \frac{r}{2} \times 2l + \pi \frac{r^2}{4}$

$= \pi r\left(l+\frac{r}{4}\right)$

Q12. A solid cylinder is melted and cast into a cone of same radius. The heights of the cone and cylinder are in the ratio

A. 9: 1

B. 1: 9

C. 3: 1

D. 1: 3

Solution:

We have radius of cylinder = radius of base of cone

$r_1 = r_2$

Let height of cylinder and cone is respectively h_1 and h^2

$= \frac{1}{3}\pi r_1 h_1 = \pi r_2 h_2$

$= \frac{h_1}{h_2} = \frac{3}{1}$

Q13. The slant height of a cone is increased by 10%. If the radius remains the same, the curved surface area is increases by

A. 10%

B. 12.1%

C. 20%

D. 21%

Solution:

Let radius of cone $= r$

Let slant height $= x$

Curved surface area $= \pi r x$

New slant height $= x + \frac{x \times 10}{100} = \frac{11x}{10}$

New volume $= \pi r \times \frac{11x}{10}$

Increase in volume $= \pi r\left(\frac{11x}{10} - x\right) = \pi r \frac{x}{10}$

Percentage increase in volume $= \frac{\frac{\pi r x}{10} \times 100}{\pi r x} = 10\%$

Q14. The height of a solid cone is 12 cm and the area of the circular base is $64\pi \text{ cm}^2$. A plane parallel to the base of the cone cuts through the cone 9 cm above the vertex of the cone, the area of the base of the new cone so formed is

A. $9\pi \text{ cm}^2$

B. $16\pi \text{ cm}^2$

C. $25\pi \text{ cm}^2$

D. $36\pi \text{ cm}^2$

Solution:

We have,

Height of cone $= 12$ cm

Area of circular base $= 64\pi \text{ cm}^2$

Height from the vertex of small cone $= 9$ cm

$\pi r^2 = 64\pi$

$r^2 = 64$

Radius of base $= 8$ cm

From similarity triangle $= \frac{12}{8} = \frac{9}{R}$

$= R = \frac{9 \times 8}{12} = 6$ cm

$R =$ radius of small cone

Area of small cone $= \pi r^2 = \pi(6)^2 = 36\pi \text{ cm}^2$

Q15. If the radius of the base of a right circular cone is $3r$ and its height is equal to the radius of the base, then its volume is

A. $\frac{1}{3}\pi r^3$

B. $\frac{2}{3}\pi r^3$

C. $3\pi r^3$
D. $9\pi r^3$
Solution:
We have,
Radius of base of cone $= 3r$
Height of cone $= 3r$
Volume of cone $= \frac{1}{3}\pi(3r)^2 \times 3r$
$= 9\pi r^3$

Q16. If the volume of two cones are in the ratio 1: 4 and their diameters are in the ratio 4: 5, then the ratio of their heights, is
A. 1: 5
B. 5: 4
C. 5: 16
D. 25: 64
Solution:
We have,
Ratio of volume of two cones $= \frac{v_1}{v_2} = \frac{1}{4}$
ratio of their diameter $= \frac{d_1}{d_2} = \frac{4}{5}$
ratio of their radius $= \frac{r_1}{r_2} = \frac{2}{\frac{5}{2}} = \frac{4}{5}$
so,
$$= \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{1}{4}$$
$$= \frac{16h_1}{25h_2} = \frac{1}{4} = \frac{h_1}{h_2} = \frac{1}{4} \times \frac{25}{16} = \frac{25}{64}$$

Q17. The curved surface area of one cone is twice that of the other while the slant height of the latter is twice that of the former. The ratio of their radii is
A. 2: 1
B. 4: 1
C. 8: 1
D. 1: 1
Solution:
We have,
$$\mathrm{A}_1 = 2\mathrm{A}_2, \frac{A_1}{A_2} = \frac{2}{1}$$
$$2l_1 = l_2, \frac{l_1}{l_2} = \frac{1}{2}$$
$$= \frac{\pi r_1 l_1}{\pi r_2 l_2} = \frac{2}{1}$$

$= \frac{r_1}{r_2} \times \frac{1}{2} = \frac{2}{1}$
$= \frac{r_1}{r_2} = \frac{4}{1}$

Q18. If the height and radius of a cone of volume V are doubled, then the volume of the cone is,
A. $3V$
B. $4V$
C. $4V$
D. 8V
Solution:
We have,
Let radius of cone $= r$
Let height of cone $= h$
Volume $= \frac{1}{3}\pi r^2 h$
New radius of cone $= 2r$
New height of cone $= 2h$
New volume $= \frac{1}{3}\pi(2r)^2 \times 2h$
$= \frac{1}{3}\pi 4r^2 \times 2h = 8\text{V}$

Q19. The ratio of the volume of a right circular cylinder and a right circular cone of the same base and height, is
A. 1: 3
B. 3: 1
C. 4: 3
D. 3: 4
Solution:
Let radius of both $= r$ cm
Let height $= h$ cm
Ratio of volume $= \frac{\pi r^2 h}{\frac{1}{3}\pi r^2 h} = \frac{1}{3}$

Q20. A right circular cylinder and a right circular cone have the same radius and the same volume. The ratio of the height of the cylinder to that of the cone is
A. 3: 5
B. 2: 5
C. 3: 1
D. 1: 3
Solution:

Let radius of both $= r$ cm
Let volume of both $=$ V cm
Ratio of height $= \frac{\pi r^2 h_1}{\frac{1}{3}\pi r^2 h_2} = \frac{v}{v}$
$= \dfrac{3h_1}{h_2} = 1$
$= \dfrac{h_1}{h_2} = \dfrac{1}{3}$

Q21. If the base radius and the height of a right circular cone are increased by 20%, then the percentage increase in volume is approximately
A. 60
B. 68
C. 73
D. 78
Solution:
Let base radius of cone $= x$
Let height of cone $= y$
Volume of cone $= \frac{1}{3}\pi x^2 y$
New radius of cone $= x + x\frac{20}{100} = \frac{6x}{5}$
New height of cone $= y + y\frac{20}{100} = \frac{6y}{5}$
New volume $= \frac{1}{3}\pi\frac{36x^2}{25} \times \frac{6y}{5} = \frac{1}{3}\pi\frac{216x^2y}{125}$
Increase in volume $= \frac{216x^2y}{125} - x^2y = \frac{91x^2y}{125}$
Percentage increase in volume $= \frac{\frac{91x^2y}{125}}{x^2y} \times 100 = 72.8\%$
Approx 73%

Q22. The diameters of two cones are equal. If their slant heights are in the ratio 5:4, the ratio of their curved surface areas, is
A. 4: 5
B. 25: 16
C. 16: 25
D. 5: 4
Solution:
We have,
Ratio of slant height of cones $= \frac{l_1}{l_2} = \frac{5}{4}$
$d_1 = d_2$
so, $r_1 = r_2$
Ratio of curved surface area $= \frac{\pi r_1 l_1}{\pi r_2 l_2} = \frac{5}{4}$

Q23. If h, S and V denote respectively the height, curved surface area and volume of a right circular cone, then $3\pi V h^3 - S^2 h^2 + 9V^2$ is equal to

A. 8
B. 0
C. 4π
D. $32\pi^2$

Solution:

We have,
$h =$ hight
$s =$ curved surface area
v = volume
Then, $3\pi v h^3 - s^2 h^2 + 9v^2$

$$= 3\pi \times \frac{1}{3}\pi r^2 h \times h^3 - \left((\pi^2 r^2 l^2)h^2\right) + 9 \times \left(\frac{1}{2}\pi r^2 h\right)^2$$

$$= \pi^2 r^2 h^4 - (\pi^2 r^2 (r^2 + h^2) h^2) + 9 \times \frac{1}{9}\pi^2 r^4 h^2$$

$$= \pi^2 r^2 h^4 - \pi^2 r^4 h^2 - \pi^2 r^2 h^4 + \pi^2 r^4 h^2 = 0$$

Q24. If a cone is cut into two parts by a horizontal plane passing through the mid-point of its axis, the ratio of the volumes of upper and lower part is

A. 1: 2
B. 2: 1
C. 1: 7
D. 1:8

Solution:

We have, (by mid point theorem)
Radius of complete cone $= 2r$
Radius of small cone after cutting $= r$
Hight of complete cone = 2 h
Hight of small cone $= h$
volume of large triangle $= \frac{1}{3}\pi(2r)^2 \times 2h$

$$= \frac{8}{3}\pi r^2 h$$

volume of small triangle $= \frac{1}{3}\pi r^2 \times h$
volume of lower part $= \left(\frac{8}{3} - \frac{1}{3}\right)\pi r^2 h = \frac{7}{3}\pi r^2 h$

So ratio of volume of lower and upper parts $= \frac{\frac{1}{3}}{\frac{7}{3}} = 1:7$

Q25. If the heights of two cones are in the ratio of 1: 4 and the radii of their bases are in the ratio 4: 1, then the ratio of their volumes is

A. 1: 2
B. 2: 3

C. 3: 4
D. 4: 1

Solution:

Given,

$$\frac{h_1}{h_2} = \frac{1}{4}$$

$$\frac{r_1}{r_2} = \frac{4}{1}$$

$$= \frac{V_1}{V_2} = \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{16}{1} \times \frac{1}{4} = \frac{4}{1}$$

$= 4: 1$