Solutions:
Grade 8 Mathematics
Chapter 3: Squares and Square Roots

Exercise 3.1

Q1. Which of the following numbers are perfect squares?

(i) 484	(ii) 625
(iii) 576	(iv) 941
(v) 961	(vi) 2500

Solution:

(i) 484

Resolving 484 into prime factors we get,

$484 = 2 \times 2 \times 11 \times 11$

Now,

Grouping the factors into pairs of equal factors, we get:

$484 = (2 \times 2) \times (11 \times 11)$

We observe that all are paired so, 484 is a perfect square.

(ii) 625

Resolving 625 into prime factors we get,

$625 = 5 \times 5 \times 5 \times 5$

Now,

Grouping the factors into pairs of equal factors, we get:

$625 = (5 \times 5) \times (5 \times 5)$

We observe that all are paired so, 625 is a perfect square.

(iii) 576

Resolving 576 into prime factors we get,

$576 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$

Now,

Grouping the factors into pairs of equal factors, we get:

$576 = (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (3 \times 3)$

We observe that all are paired so, 576 is a perfect square.

(iv) 941

Resolving 941 into prime factors we get,

$941 = 941 \times 1$

Now, as 941 itself is a prime number.
Hence, it do not have a perfect square.

(v) 961
Resolving 961 into prime factors we get,
$961 = 31 \times 31$
Now,
Grouping the factors into pairs of equal factors, we get:
$961 = (31 \times 31)$
We observe that all are paired so, 961 is a perfect square.

(vi) 2500
Resolving 2500 into prime factors we get,
$2500 = 2 \times 2 \times 5 \times 5 \times 5 \times 5$
Now,
Grouping the factors into pairs of equal factors, we get:
$2500 = (2 \times 2) \times (5 \times 5) \times (5 \times 5)$
We observe that all are paired so, 2500 is a perfect square.

Q2. Show that each of the following numbers is a perfect square. Also find the number whose square is the given number in each case:
(i) 1156
(ii) 2025
(iii)14641
(iv) 4761

Solution:
(i) 1156
Resolving 1156 into prime factors we get,
$1156 = 2 \times 2 \times 17 \times 17$
Now, grouping the factors into pairs of equal factors, we get,
$1156 = (2 \times 2) \times (17 \times 17)$
As all factors are paired.
Hence, 1156 is a perfect square.
Again,
$1156 = (2 \times 17) \times (2 \times 17)$
$= 34 \times 34$
$= (34)^2$
Thus, 1156 is a square of 34.

(ii) 2025

Resolving 2025 into prime factors we get,
$2025 = 3 \times 3 \times 3 \times 3 \times 5 \times 5$
Now, grouping the factors into pairs of equal factors, we get,
$2025 = (3 \times 3) \times (3 \times 3) \times (5 \times 5)$
As all factors are paired.
Hence, 2025 is a perfect square.
Again,
$2025 = (3 \times 3 \times 5) \times (3 \times 3 \times 5)$
$= 45 \times 45$
$= (45)^2$
Thus, 2025 is a square of 45.

(iii)14641
Resolving 14641 into prime factors we get,
$14641 = 11 \times 11 \times 11 \times 11$
Now, grouping the factors into pairs of equal factors, we get,
$14641 = (11 \times 11) \times (11 \times 11)$
As all factors are paired.
Hence, 14641 is a perfect square.
Again,
$14641 = (11 \times 11) \times (11 \times 11)$
$= 121 \times 121$
$= (121)^2$
Thus, 14641 is a square of 121.

(iv) 4761
Resolving 4761 into prime factors we get,
$4761 = 3 \times 3 \times 23 \times 23$
Now, grouping the factors into pairs of equal factors, we get,
$4761 = (3 \times 3) \times (23 \times 23)$
As all factors are paired.
Hence, 4761 is a perfect square.
Again,
$4761 = (3 \times 23) \times (3 \times 23)$
$= 69 \times 69$
$= (69)^2$
Thus, 4761 is a square of 69.

Q3. Find the smallest number by which the given number must be multiplied so that the product is a perfect square:

(i) 23805
(ii) 12150
(iii) 7688

Solution:

(i) 23805

Resolving 23805 into prime factors, we get:

$23805 = 3 \times 3 \times 23 \times 23 \times 5$

Obtained factors can be paired into equal factors except for 5.

To pair it equally multiply with 5.

$23805 \times 5 = 3 \times 3 \times 5 \times 5 \times 23 \times 23$

Again,

$23805 \times 5 = (3 \times 5 \times 23) \times (3 \times 5 \times 23)$

$= 345 \times 345$

$= (345)^2$

Therefore, product is the square of 345.

(ii) 12150

Resolving 12150 into prime factors, we get:

$12150 = 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5$

The obtained factors can be paired into equal factors, except for a 2 and a 3.

To pair it equally multiply with 2 and 3.

$12150 \times 2 \times 3 = 2 \times \mathbf{2} \times \mathbf{3} \times 3 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5$

Again,

$12150 \times 2 \times 3 = (2 \times 2) \times (3 \times 3 \times 3 \times 3 \times 3 \times 3) \times (5 \times 5)$

$= (2^2) \times (3^6) \times (5^2)$

$= (2 \times 3 \times 3 \times 3 \times 5)^2$

$= (270)^2$

Therefore, the product is the square of 270.

(iii) 7688

Resolving 7688 into prime factors, we get,

$7688 = 2 \times 2 \times 31 \times 31 \times 2$

Obtained factors can be paired into equal factors except for 2.

To pair it equally multiply with 2.

$7688 \times 2 = 2 \times 2 \times 2 \times 2 \times 31 \times 31$

Again,

$7688 \times 2 = (2 \times 2 \times 31) \times (2 \times 2 \times 31)$

$= 124 \times 124$

$= (124)^2$

Therefore, product is the square of 124.

Q4. Find the smallest number by which the given number must be divided so that the resulting number is a perfect square:

(i) 14283

(ii) 1800

(iii) 2904

Solution:

(i) 14283

Resolving 14283 into prime factors, we get:

$14283 = 3 \times 3 \times 3 \times 23 \times 23$

Obtained factors can be paired into equal factors except for 3.

So, eliminate 3 by diving the number with 3.

$\frac{14283}{3} = (3 \times 3) \times (23 \times 23)$

Again,

$\frac{14283}{3} = (3 \times 23) \times (3 \times 23)$

$= 69 \times 69$

$= (69)^2$

Therefore,

The resultant is the square of 69.

(ii) 1800

Resolving 1800 into prime factors, we get:

$1800 = 2 \times 2 \times 5 \times 5 \times 3 \times 3 \times 2$

Obtained factors can be paired into equal factors except for 2.

So, eliminate 2 by dividing the number with 2.

$\frac{1800}{2} = (2 \times 2) \times (3 \times 3) \times (5 \times 5)$

Again,

$\frac{1800}{2} = (2 \times 3 \times 5) \times (2 \times 3 \times 5)$

$= 30 \times 30$

$= (30)^2$

Therefore, the resultant is the square of 30.

(iii) 2904

Resolving 2904 into prime factors, we get:

$2904 = 2 \times 2 \times 11 \times 11 \times 2 \times 3$

Obtained factors can be paired into equal factors except for 2 and 3.

So, eliminate 6 by diving the dividing the number with 6.

$\frac{2904}{6} = (2 \times 2) \times (11 \times 11)$

Again,

$\frac{2904}{6} = (2 \times 11) \times (2 \times 11)$

$= 22 \times 22$

$= (22)^2$

Therefore, the resultant is the square of 22.

Q5. Which of the following numbers are perfect squares?

11, 12, 16, 32, 36, 50, 64, 79, 81, 111, 121

Solution:

11: Since 11 is a prime number, hence, it is not a perfect square.

12: Since, 12 is ending with 2, hence, it is .not a perfect square

16: Since, $16 = 4 \times 4$

$= (16)^2$

Therefore, it is a perfect square.

32: Since, 32 is ending with 2, hence, it is not a perfect square.

36: Since, $36 = 6^2$

Hence, it is a perfect square.

50: Since, $50 = 5^2 \times 2$

Hence, it is not a perfect square.

64: Since, $64 = 8^2$

Hence, it is a perfect square.

79: Since it is a prime number so it cannot be a perfect square.

81: Since, $81 = 9^2$

Hence, it is a perfect square.

111: Since, 111 is a prime number so it cannot be a perfect square.

121: Since, $121 = 11^2$

Hence, it is perfect square.

Q6. Using prime factorization method, find which of the following numbers are perfect squares? 189, 225, 2048, 343, 441, 2961, 11025, 3549

Solution:

Since,

$189 = 3^2 \times 3 \times 7$

It cannot be written as pair of two equal factors, so 189 is not a perfect square.

Since,

$225 = (5 \times 5) \times (3 \times 3)$

It can be written as pair of two equal factors, so 225 is a perfect square.
Since,
$2048 = (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times 2$
All the factors cannot be written as pair of two equal factors, so 2048 is not a perfect square.
Since,
$343 = (7 \times 7) \times 7$
It cannot be written as pair of two equal factors, so 343 is not a perfect square.
Since,
$441 = (7 \times 7) \times (3 \times 3)$
It can be written as pair of two equal factors, so 441 is a perfect square.
Since,
$2916 = (3 \times 3) \times (3 \times 3) \times (3 \times 3) \times (2 \times 2)$
It can be written as pair of two equal factors, so 2916 is a perfect square.
Since,
$11025 = (5 \times 5) \times (3 \times 3) \times (7 \times 7)$
It can be written as pair of two equal factors, so 11025 is a perfect square.
Since,
$3549 = (13 \times 13) \times 3 \times 7$
It cannot be written as pair of two equal factors, so 3549 is not a perfect square.

Q7. By what number should each of the following numbers multiply to get a perfect square in each case? Also find the number whose square is the new number.
(i) 8820 (ii) 3675
(iii) 605 (iv) 2880
(v) 4056 (vi) 3468
(vii) 7776

Solution:
(i) 8820
$8820 = (2 \times 2) \times (3 \times 3) \times (7 \times 7) \times 5$
In the above factors only 5 is unpaired.
So, multiply the number with 5 to make it paired.
Again,
$8820 \times 5 = 2 \times 2 \times 3 \times 3 \times 7 \times 7 \times 5 \times 5$
$= (2 \times 2) \times (3 \times 3) \times (7 \times 7) \times (5 \times 5)$
$= (2 \times 3 \times 7 \times 5) \times (2 \times 3 \times 7 \times 5)$
$= 210 \times 210$
$= (210)^2$
So, the product is the square of 210.

(ii) 3675

$3675 = (5 \times 5) \times (7 \times 7) \times 3$

In the above factors only 3 is unpaired.

So, multiply the number with 3 to make it paired.

Again,

$3675 \times 3 = 5 \times 5 \times 7 \times 7 \times 3 \times 3$

$= (5 \times 5) \times (7 \times 7) \times (3 \times 3)$

$= (3 \times 5 \times 7) \times (3 \times 5 \times 7)$

$= 105 \times 105$

$= (105)^2$

So, the product is the square of 105.

(iii) 605

$605 = 5 \times (11 \times 11)$

In the above factors only 5 is unpaired.

So, multiply the number with 5 to make it paired.

Again,

$605 \times 5 = 5 \times 5 \times 11 \times 11$

$= (5 \times 5) \times (11 \times 11)$

$= (5 \times 11) \times (5 \times 11)$

$= 55 \times 55$

$= (55)^2$

So, the product is the square of 55.

(iv) 2880

$2880 = 5 \times (3 \times 3) \times (2 \times 2) \times (2 \times 2) \times (2 \times 2)$

In the above factors only 5 is unpaired.

So, multiply the number with 5 to make it paired.

Again,

$2880 \times 5 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$

$= (2 \times 2) \times (2 \times 2) \times (2 \times 2)(3 \times 3) \times (5 \times 5)$

$= (2 \times 2 \times 2 \times 3 \times 5) \times (2 \times 2 \times 2 \times 3 \times 5)$

$= 120 \times 120$

$= (120)^2$

So, the product is the square of 120.

(v) 4056

$4056 = (2 \times 2) \times (13 \times 13) \times 2 \times 3$

In the above factors only 2 and 3 are unpaired.

So, multiply the number with 6 to make it paired.

Again,

$4056 \times 6 = 2 \times 2 \times 13 \times 13 \times 2 \times 2 \times 3 \times 3$

$= (2 \times 2) \times (13 \times 13) \times (2 \times 2)(3 \times 3)$

$= (2 \times 2 \times 3 \times 13) \times (2 \times 2 \times 3 \times 13)$

$= 156 \times 156$

$= (156)^2$

So, the product is the square of 156.

(vi) 3468

$3468 = (2 \times 2) \times 3 \times (17 \times 17)$

In the above factors only 3 are unpaired.

So, multiply the number with 3 to make it paired.

$3468 \times 3 = (2 \times 2) \times (3 \times 3) \times (17 \times 17)$

$= (2 \times 3 \times 17) \times (2 \times 3 \times 17)$

$= 102 \times 102$

$= (102)^2$

So, the product is the square of 102.

(vii) 7776

$7776 = (2 \times 2) \times (2 \times 2) \times (3 \times 3) \times (3 \times 3) \times 2 \times 3$

In the above factors only 2 and 3 are unpaired.

So, multiply the number with 6 to make it paired.

Again,

$7776 \times 6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$= (2 \times 2) \times (2 \times 2) \times (2 \times 2)(3 \times 3) \times (3 \times 3) \times (3 \times 3)$

$= (2 \times 2 \times 2 \times 3 \times 3 \times 3) \times (2 \times 2 \times 2 \times 3 \times 3 \times 3)$

$= 216 \times 216$

$= (216)^2$

So, the product is the square of 216.

Q8. By What numbers should each of the following be divided to get a perfect square in each case? Also, find the number whose square is the new number.

(i) 16562

(ii) 3698

(iii) 5103

(iv) 3174

(v) 1575

Solution:

(i) 16562

$16562 = (7 \times 7) \times (13 \times 13) \times 2$

$\frac{16562}{2} = (7 \times 7) \times (13 \times 13)$

$\frac{16562}{2} = (7 \times 13) \times (7 \times 13)$

$= 91 \times 91$

$= 91^2$

Therefore, the resultant is the square of 91.

(ii) 3698

$3698 = 2 \times (43 \times 43)$

$\frac{3699}{2} = 43 \times 43$

$= 43^2$

Therefore, the numbers must be divided by 2 and resultant is square of 43.

(iii) 5103

$5103 = (3 \times 3) \times (3 \times 3) \times (3 \times 3) \times 7$

$\frac{5103}{7} = (3 \times 3 \times 3) \times (3 \times 3 \times 3)$

$= 27 \times 27$

$= 27^2$

Therefore, the number must be divided by 7 and resultant is square of 27.

(iv) 3174

$3174 = 2 \times 3 \times (23 \times 23)$

$\frac{3174}{6} = 23 \times 23$

$= 23^2$

Therefore, the number must be divided by 6 and the resultant is square of 23.

(v) 1575

$1575 = 3 \times 3 \times 5 \times 5 \times 7$

$\frac{1575}{7} = 3 \times 3 \times 5 \times 5$

$= (3 \times 5) \times (3 \times 5)$

$= 15 \times 15$

$= 15^2$

Therefore, the number must be divided by 7 and the resultant is square of 15.

Q9. Find the greatest number of two digits which is a perfect square.

Solution:

The greatest two-digit number is 99.

The largest perfect square within this range is 81 (since $9^2 = 81$).

Thus, the greatest two-digit perfect square is 81.

Q10. Find the least number of three digits which is perfect square.

Solution:

The smallest three-digit number is 100.

Checking for the smallest perfect square: $\sqrt{100} = 10$

Since $10^2 = 100$, the least three-digit number is a perfect square is 100.

Q11. Find the smallest number by which 4851 must be multiplied so that the product becomes a perfect square.

Solution:

Factors of 4851 are:

$4851 = 3 \times 3 \times 7 \times 7 \times 11$

Pairs $= 3^2 \times 7^2$

Hence, 4851 should be multiplied by 11 in order to get a perfect square when smallest number multiplied to 4851.

Q12. Find the smallest number by which 28812 must be divided so that it becomes a perfect square. Also find the number whose square is the resulting number.

Solution:

Factors of 28812 are:

$28812 = 2 \times 2 \times 3 \times 7 \times 7 \times 7 \times 7$

Pairs $= 2^2 \times 7^2 \times 7^2$

Hence, 28812 should be divided by 3 in order to get a perfect square when divided by the least number.

The square root will be: $2 \times 7 \times 7 = 98$

Q13. Find the smallest number by which 1152 must be divided so that it becomes a perfect square. Also find the number whose square is the resulting number.

Solution:

Factors of 1152 are:

$1152 = 2^7 \times 3^2$

Pairs $= 2^6 \times 3^2$

Hence, 1152 should be divided by 2 in order to get the perfect square.

Hence the number after division by $2 = \frac{1152}{2} = 576$

Factors of 576 are $= 2^6 \times 3^2 = 24^2$

Hence, resulting number is the square of 24.

Exercise 3.2

Q1. The following numbers are not perfect squares. Give reason.

(i) 1547

(ii) 45743

(iii)8948

(iv) 333333

Solution:

Numbers ending with 2, 3, 7 or 8 are not perfect squares. So,

(i) 1547 is not perfect square

(ii) 45743 is not perfect square

(iii) 8948 is not perfect square

(iv) 333333 is not perfect square

Q2. Show that the following numbers are not perfect squares:

(i) 9327 (ii) 4058

(iii)22453 (iv) 743522

Solution:

From the given number, 7, 8, 3, 2 as ending numbers respectively.

As mentioned above ending with 2, 3, 7, 8 are not perfect square.

So, these given numbers are not perfect squares.

Q3. The square of which of the following numbers would be an old number?

(i) 731 (ii) 3456

(iii)5559 (iv) 42008

Solution:

Square of an odd number is an odd number.

Square of an even number is an even number.

(i) 731: It is an odd number so its square is also odd number.

(ii) 3456: It is an even number so its square is also even number.

(iii) 5559: It is an odd number so its square is also odd number.

(iv) 42008: It is an even number so its square is also even number.

Q4. What will be the units digit of the squares of the following numbers?

(i) 52 (ii) 977

(iii) 4583 (iv) 78367

(v) 52698 (vi) 99880

(vii) 12796 (viii) 55555

(ix) 53924

Solution:

(i) 52

Unit digit of $(52)^2$ = unit digit of $(2)^2 = 4$

(ii) 977

Unit digit of $(977)^2$ = unit digit of $(7)^2 = 9$

(iii) 4583

Unit digit of $(4583)^2$ = unit digit of $(3)^2 = 9$

(iv) 78367

Unit digit of $(78367)^2$ = unit digit of $(7)^2 = 9$

(v) 52698

Unit digit of $(52698)^2$ = unit digit of $(8)^2 = 4$

(vi) 99880

Unit digit of $(99880)^2$ = unit digit of $(0)^2 = 0$

(vii) 12796

Unit digit of $(12796)^2$ = unit digit of $(6)^2 = 6$

(viii) 55555

Unit digit of $(55555)^2$ = unit digit of $(5)^2 = 5$

(ix) 53924

Unit digit of $(53924)^2$ = unit digit of $(4)^2 = 6$

Q5. Observe the following pattern.

$1 + 3 = 2$

$1 + 3 + 5 = 3^2$

$1 + 3 + 5 + 7 = 4^2$

Find the value of $1 + 3 + 5 + 7 + 9 + \cdots$ up to n terms.

Solution:

The pattern here is the square of the number on the Right-hand side is equal to the sum of all the numbers on the left-hand side.

Thus, for n terms,

$1 + 3 + 5 + \cdots..n$ terms $= n^2$ [As there are n terms]

Q6. Observe the following pattern:

$2^2 - 1^2 = 2 + 1$

$3^2 - 2^2 = 3 + 2$

$4^2 - 3^2 = 4 + 3$

$5^2 - 4^2 = 5 + 4$

And find the value of
(i) $100^2 - 99^2$
(ii) $111^2 - 109^2$
(iii) $99^2 - 96^2$

Solution:

(i) $100^2 - 99^2$
$= 100 + 99$
$= 199$
(ii) $111^2 - 109^2$
$= 111^2 - 110^2 + 110^2 - 109^2$
$= (111 + 110) + (110 + 109)$
$= 440$
(iii) $99^2 - 96^2$
$= 99^2 - 98^2 + 98^2 - 97^2 + 97^2 - 96^2$
$= (99 + 98) + (98 + 92) + (97 + 96)$
$= 585$

Q7. Which of the following triplets are Pythagorean?
(i) (8,15,17)
(ii) (18,80,82)
(iii) (14,48,51)
(iv) (10,24,26)
(v) (16,63,65)
(vi) (12,35,38)

Solution:

(i) (8,15,17)
L.H.S $= 8^2 + 15^2 = 289$
R.H.S $= 17^2 = 289$
L.H.S = R.H.S
So, it is Pythagoras.

(ii) (18,80,82)
L.H.S $= 18^2 + 80^2 = 6724$
R.H.S $= 82^2 = 6724$
L.H.S = R.H.S
So, it is Pythagoras.

(iii) (14,48,51)
L.H.S $= 14^2 + 48^2 = 2500$

$\text{R.H.S} = 51^2 = 2601$
L.H.S ≠ R.H.S
So, it is not Pythagoras.

(iv) (10,24,26)
$\text{L.H.S} = 10^2 + 24^2 = 676$
$\text{R.H.S} = 26^2 = 676$
L.H.S = R.H.S
So, it is Pythagoras.

(v) (16,63,65)
$\text{L.H.S} = 16^2 + 63^2 = 4225$
$\text{R.H.S} = 65^2 = 4225$
L.H.S = R.H.S
So, it is Pythagoras.

(vi) (12,35,38)
$\text{L.H.S} = 12^2 + 35^2 = 1369$
$\text{R.H.S} = 38^2 = 1444$
L.H.S ≠ R.H.S
So, it is Pythagoras.

Q8. Observe the following pattern:

$$(1 \times 2) + (2 \times 3) = \frac{2 \times 3 \times 4}{3}$$

$$(1 \times 2) + (2 \times 3) + (3 \times 4) = \frac{3 \times 4 \times 5}{3}$$

$$(1 \times 2) + (2 \times 3) + (3 \times 4) + (4 \times 5) = \frac{4 \times 5 \times 6}{3}$$

And find the value of

$(1 \times 2) + (2 \times 3) + (3 \times 4) + (4 \times 5)(5 \times 6)$

Solution:

From observation:

$$(1 \times 2) + (2 \times 3) + (3 \times 4) + (4 \times 5) + (5 \times 6) = \frac{5 \times 6 \times 7}{3} = 70$$

Q9. Observe the following pattern:

$$1 = \frac{1}{2}\{1 \times (1 + 1)\}$$

$$1 + 2 = \frac{1}{2}\{2 \times (2 + 1)\}$$

$$1 + 2 + 3 = \frac{1}{2}\{3 \times (3 + 1)\}$$

$$1 + 2 + 3 + 4 = \frac{1}{2}\{4 \times (4 + 1)\}$$

And find the values of each of the following:

(i) $1 + 2 + 3 + 4 + 5 + \cdots + 50$

(ii) $31 + 32 + \cdots + 50$

Solution:

R.H.S $= \frac{1}{2}$[No. of terms in L.H.S × (No. of terms + 1)] (Therefore, only when L.H.S starts with 1)

Therefore,

(i) $1 + 2 + 3 + \cdots .50 = \frac{1}{2}[50 \times (50 + 1)]$

$= 25 \times 51$

$= 1275$

(ii) $31 + 32 + \cdots . +50 = (1 + 2 + 3 + \cdots + 50) - (1 + 2 + \cdots .30)$

$= 1275 - \left[\frac{1}{2}(30 \times 30 + 1)\right]$

$= 1275 - 465$

$= 810$

Q10. Observe the following pattern:

$$1^2 = \frac{1}{6}[1 \times (1 + 1) \times (2 \times 1 + 1)]$$

$$1^2 + 2^2 = \frac{1}{2}[2 \times (2 + 1) \times (2 \times 2 + 1)]$$

$$1^2 + 2^2 + 3^2 = \frac{1}{6}[3 \times (3 + 1) \times (2 \times 3 + 1)]$$

$$1^1 + 2^2 + 3^2 + 4^2 = \frac{1}{6}[4 \times (4 + 1) \times (2 \times 4 + 1)]$$

And find the values of each of the following.

(i) $1^2 + 2^2 + 3^2 + 4^2 + \cdots + 10^2$

(ii) $5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2 + 11^2 + 12^2$

Solution:

R.H.S $= \frac{1}{6}$[(No. of terms in L.H.S) × (No. +1) × (2 × No. + 1)]

(i) $1^2 + 2^2 + 3^2 + 4^2 + \cdots ... + 10^2 = \frac{1}{6}[10(10 + 1) \times (2 \times 10 + 1)]$

$= \frac{1}{6}[2310]$

$= 385$

(ii) $5^2 + 6^2 + \cdots .. + 12^2 = 1^2 + 2^2 + \cdots .. 12^2 - (1^2 + 2^2 + 3^3 + 4^2)$

$= \frac{1}{6}[12 \times (12 + 1) \times (2 \times 12 + 1)] - \frac{1}{6}[4 \times (4 + 1) \times (2 \times 4 + 1)]$

$= 650 - 30$

$= 620$

Q11. Which of the following numbers are squares of even numbers?

121, 225, 256, 324, 1296, 6561, 5476, 4489, 373758

Solution:

Only even numbers be the square of even numbers.

So, 256, 324, 1296, 5476, 373758 can be square of even numbers but 373758 is not a perfect square.

So, 256, 324, 1296, 5476 are the required number.

Q12. By just examining the unit digits, can you tell which of the following cannot be whole squares?

(i) 1026 (ii) 1028

(iii)1024 (iv) 1022

(v) 1023 (vi) 1027

Solution:

Numbers ending with 2, 3, 7, 8 cannot be perfect square.

So, (ii) 1028 (iv) 1022 (v) 1023 (vi) 1027 cannot be whole squares.

Q13. Which of the numbers for which you cannot decide whether they are squares.

Solution:

A natural number cannot be a perfect square if its unit digit is 2, 3, 7, or 8. However, numbers ending in 0, 1, 4, 5, 6, or 9 may or may not be perfect squares.

Q14. Write five numbers which you cannot decide whether they are square just by looking at the unit's digit.

Solution:

Any natural number ending with 0, 1, 4, 5, 6 or 9 can be or cannot be a square number.

Hence, the five examples are:

(i) 2061

The ending digit is 1. Hence, it may or may not be a square number.

(ii) 1069

The ending digit is 9. Hence, it may or may not be a square number.

(iii) 1234

The ending digit is 4. Hence, it may or may not be a square number.

(iv) 56790
The ending digit is 0. Hence, it may or may not be a square number.
(v) 76555
The ending digit is 5. Hence, it may or may not be a square number.

Q15. Write true (T) or false (F) for the following statements.
(i) The number of digits in a square number is even.
(ii) The square of a prime number is prime.
(iii) The sum of two square numbers is a square number.
(iv) The difference of two square numbers is a square number.
(v) The product of two square numbers is a square number.
(vi) No square number is negative.
(vii) There is no square number between 50 and 56 .
(viii) There are fourteen square number up to 200.
Solution:
(i) False: Because 169 is square number with odd digit.
(ii) False: Square of 3 (Prime) is 9 (not prime).
(iii) False: Sum of 2^2 and 3^2 is 13 which is not square number.
(iv) False: Difference of 3^2 and 2^2 is 5 , which is not square number.
(v) True: Because the square of 2^2 and 3^2 is 36 which is square of 6.
(vi) True: As $(-2)^2$ is 4 , i.e. not negative.
(vii) True: As there is no square number between them.
(viii) True: The fourteen numbers upto 200 are: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196

Exercise 3.3

Q1. Find the squares of the following numbers using column method. Verify the result by finding the square using the usual multiplication:
(i) 25
(ii) 37
(iii) 54
(iv) 71
(v) 96
Solution:
(i) 25
Here, $a = 2, b = 5$

Column 1	Column 2	Column 3
a^2	$2ab$	b^2

4	20	$2\underline{5}$
+2	+2	
$\underline{6}$	$2\underline{2}$	
6	2	5

$25^2 = 625$
And,
$25^2 = 25 \times 25 = 625$

(ii) 37
Here, $a = 3, b = 7$

Column 1	Column 2	Column 3
a^2	$2ab$	b^2
9	42	$4\underline{9}$
+4	+4	
$\underline{13}$	$4\underline{6}$	
13	6	9

$37^2 = 1369$
And
$37^2 = 37 \times 37 = 1369$

(iii) 54
Here, $a = 5, b = 4$

Column 1	Column 2	Column 3
a^2	$2ab$	b^2
25	40	$1\underline{6}$
+4	+1	
$\underline{29}$	$4\underline{1}$	
29	1	6

$54^2 = 2916$

And
$54^2 = 54 \times 54 = 2916$

(iv) 71
Here, $a = 7, b = 1$

Column 1	Column 2	Column 3
a^2	$2ab$	b^2
49	14	$0\underline{1}$
1	0	
$\underline{49}$	$1\underline{4}$	
50	4	1

$71^2 = 5041$
And
$71^2 = 71 \times 71 = 5041$

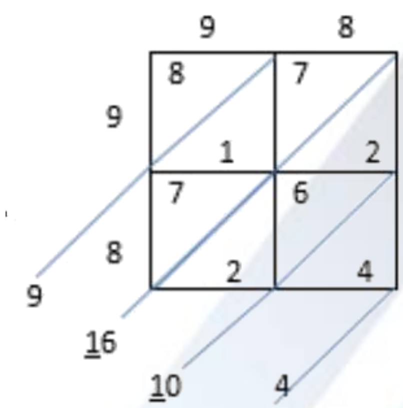
(v) 96
Here, $a = 9, b = 6$

Column 1	Column 2	Column 3
a^2	$2ab$	b^2
81	108	$3\underline{6}$
11	3	
$\underline{92}$	$11\underline{1}$	
92	1	6

$96^2 = 9216$
And
$96^2 = 96 \times 96 = 9216$

Q2. Find the squares of the following numbers using diagonal method:
(i) 98
(ii) 273
(iii)348

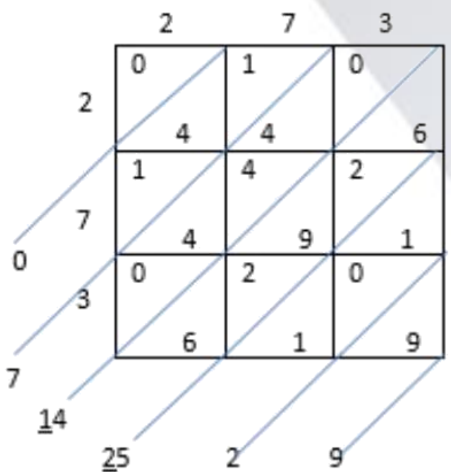
(iv) 295
(v) 171
Solution:
(i) 98
Step I: Obtain the number and count the number of digits in it. Let there be n digits in the number to be squared.
Step II: Draw square and divide it into n^2 sub-squares of the same size by drawing $(n-1)$ horizontal and $(n-1)$ vertical lines.
Step III: Draw the diagonals of each sub-square.
Step IV: Write the digits of the number to be squared along left vertical side sand top horizontal side of the squares as shown below.
Step V: Multiply each digit on the left of the square with each digit on top of the column one-by-one. Write the units digit of the product below the diagonal and tens digit above the diagonal of the corresponding square.
Step VI: Starting below the lowest diagonal sum the digits along the diagonals so obtained. Write the units digit of the sum and take carry, the tens digit (if any) to the diagonal above.
Step VII: Obtain the required square by writing the digits from the left-most side.



$(98)^2=9604$

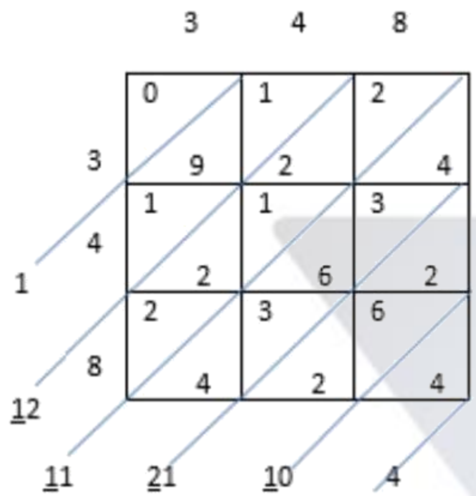
(ii) 273
Step I: Obtain the number and count the number of digits in it. Let there be n digits in the number to be squared.
Step II: Draw square and divide it into n^2 sub-squares of the same size by drawing $(n-1)$ horizontal and $(n-1)$ vertical lines.
Step III: Draw the diagonals of each sub-square.
Step IV: Write the digits of the number to be squared along left vertical side sand top horizontal side of the squares as shown below.

Step V: Multiply each digit on the left of the square with each digit on top of the column one-by-one. Write the units digit of the product below the diagonal and tens digit above the diagonal of the corresponding square.
Step VI: Starting below the lowest diagonal sum the digits along the diagonals so obtained. Write the units digit of the sum and take carry, the tens digit (if any) to the diagonal above.
Step VII: Obtain the required square by writing the digits from the left-most side.



$(273)^2 = 74529$

(iii) 348
Step I: Obtain the number and count the number of digits in it. Let there be n digits in the number to be squared.
Step II: Draw square and divide it into n^2 sub-squares of the same size by drawing $(n-1)$ horizontal and $(n-1)$ vertical lines.
Step III: Draw the diagonals of each sub-square.
Step IV: Write the digits of the number to be squared along left vertical side sand top horizontal side of the squares as shown below.
Step V: Multiply each digit on the left of the square with each digit on top of the column one-by-one. Write the units digit of the product below the diagonal and tens digit above the diagonal of the corresponding square.
Step VI: Starting below the lowest diagonal sum the digits along the diagonals so obtained. Write the units digit of the sum and take carry, the tens digit (if any) to the diagonal above.
Step VII: Obtain the required square by writing the digits from the left-most side.



$348^2 = 121104$

(iv) 295

Step I: Obtain the number and count the number of digits in it. Let there be n digits in the number to be squared.

Step II: Draw square and divide it into n^2 sub-squares of the same size by drawing $(n-1)$ horizontal and $(n-1)$ vertical lines.

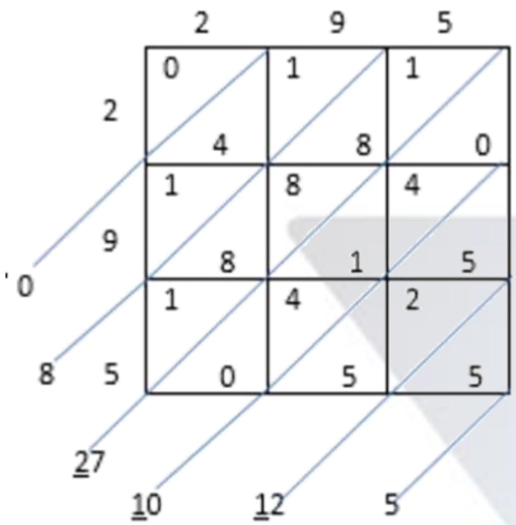
Step III: Draw the diagonals of each sub-square.

Step IV: Write the digits of the number to be squared along left vertical side sand top horizontal side of the squares as shown below.

Step V: Multiply each digit on the left of the square with each digit on top of the column one-by-one. Write the units digit of the product below the diagonal and tens digit above the diagonal of the corresponding square.

Step VI: Starting below the lowest diagonal sum the digits along the diagonals so obtained. Write the units digit of the sum and take carry, the tens digit (if any) to the diagonal above.

Step VII: Obtain the required square by writing the digits from the left-most side. 0



$(295)^2 = 87025$

(v) 171

Step I: Obtain the number and count the number of digits in it. Let there be n digits in the number to be squared.

Step II: Draw square and divide it into n^2 sub-squares of the same size by drawing $(n-1)$ horizontal and $(n-1)$ vertical lines.

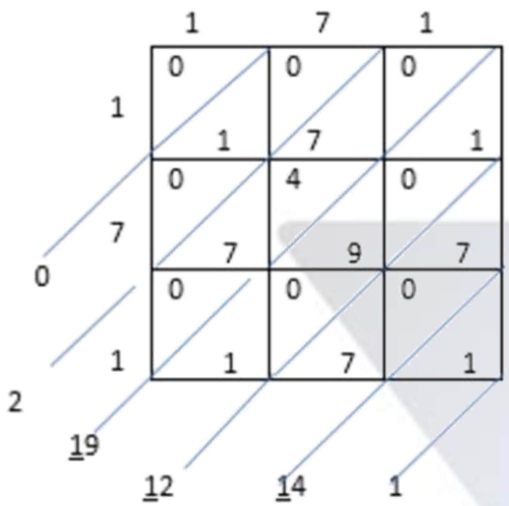
Step III: Draw the diagonals of each sub-square.

Step IV: Write the digits of the number to be squared along left vertical side sand top horizontal side of the squares as shown below.

Step V: Multiply each digit on the left of the square with each digit on top of the column one-by-one. Write the units digit of the product below the diagonal and tens digit above the diagonal of the corresponding square.

Step VI: Starting below the lowest diagonal sum the digits along the diagonals so obtained. Write the units digit of the sum and take carry, the tens digit (if any) to the diagonal above.

Step VII: Obtain the required square by writing the digits from the left-most side.



$(171)^2 = 29241$

Q3. Find the squares of the following numbers:

(i) 127 (ii) 503
(iii) 450 (iv) 862
(v) 265

Solution:

(i) $(127)^2 = 127 \times 127 = 16129$
(ii) $(503)^2 = 503 \times 503 = 253009$
(iii) $(451)^2 = 451 \times 451 = 203401$
(iv) $(862)^2 = 862 \times 862 = 743044$
(v) $(265)^2 = 265 \times 265 = 70225$

Q4. Find the squares of the following numbers:

(i) 425 (ii) 575
(iii)405 (iv) 205
(v) 95 (vi) 745
(vii) 512 (viii) 995

Solution:

(i) 425
The square of 425 is:
$(425)^2 = 425 \times 425 = 180625$
Hence, the square of 425 is 180625.
(ii) 575
The square of 575 is:
$(575)^2 = 575 \times 575 = 330625$

Hence, the square of 575 is 330625.
(iii) 405
The square of 405 is:
$(405)^2 = 405 \times 405 = 164025$
Hence, the square of 405 is 164025.
(iv) 205
The square of 205 is:
$(205)^2 = 205 \times 205 = 42025$
Hence, the square of 205 is 42025.
(v) 95
The square of 95 is:
$(95)^2 = 95 \times 95 = 9025$
Hence, the square of 95 is 9025.
(vi) 745
The square of 745 is:
$(745)^2 = 745 \times 745 = 555025$
Hence, the square of 745 is 555025.
(vii) 512
The square of 512 is:
$(512)^2 = 512 \times 512 = 262144$
Hence, the square of 512 is 262144.
(viii) 995
The square of 995 is:
$(995)^2 = 995 \times 995 = 990025$
Hence, the square of 995 is 990025.

Q5. Find the squares of the following numbers using the identify $(a + b)^2 = a^2 + 2ab + b^2$:
(i) 405
(ii) 510
(iii) 1001
(iv) 209
(v) 605

Solution:
(i) 405
We have,
$(405)^2 = (400 + 5)^2$
$= (400)^2 + 5^2 + 2(400)(5)$
$= 160000 + 25 + 4000$
$= 164025$

(ii) 510
We have,
$(510)^2 = (500 + 10)^2$
$= 250000 + 100 + 10000$
$= 260100$

(iii) 1001
We have,
$(1001)^2 = (1000 + 1)^2$
$= (1000)^2 + 1 + 2(1000)$
$= 1000000 + 1 + 2000$
$= 1002001$

(iv) 209
We have,
$(209)^2 = (200 + 9)^2$
$= (200)^2 + 9^2 + 2(200)(9)$
$= 40000 + 81 + 3600$
$= 43681$

(v) 605
We have,
$(605)^2 = (600 + 5)^2$
$= (600)^2 + 5^2 + 2(600)(5)$
$= 360000 + 25 + 6000$
$= 366025$

Q6. Find the squares of the following numbers using the identity $(a - b)^2 = a^2 - 2ab + b^2$:
(i) 395 (ii) 995
(iii)495 (iv) 498
(v) 99 (vi) 999
(vii) 599

Solution:
(i) 395
$395 = (400 - 5)^2$
$= (400)^2 + 5^2 - 2(400)(5)$
$= 160000 + 25 - 4000$
$= 156025$

(ii) 995

$995 = (1000 - 5)^2$
$= (1000)^2 + 5^2 - 2(1000)(5)$
$= 1000000 + 25 - 10000$
$= 990025$

(iii)495
$495 = (500 - 5)^2$
$= (500)^2 + 5^2 - 2(500)(5)$
$= 250000 + 25 - 5000$
$= 245025$

(iv) 498
$498 = (500 - 2)^2$
$= (500)^2 + 2^2 - 2(500)(2)$
$= 250000 + 4 - 2000$
$= 248004$

(v) 99
$99 = (100 - 1)^2$
$= (100)^2 + 1^2 - 2(100)(1)$
$= 10000 + 1 - 200$
$= 9799$

(vi) 999
$999 = (1000 - 1)^2$
$= (1000)^2 + 1^2 - 2(1000)(1)$
$= 1000000 + 1 - 2000$
$= 998001$

(vii)599
$(600 - 1)^2$
$= (600)^2 + 1^2 - 2(600)(1)$
$= 360000 + 1 - 1200$
$= 358801$

Q7. Find the squares of the following numbers by visual method:
(i) 52 (ii) 95
(iii) 505 (iv) 702
(v) 99

Solution:

(i) 52
$(52)^2 = (50 + 2)^2$
$= 50^2 + 2^2 + (2 \times 50 \times 2)$
$= 2500 + 4 + 200$
$= 2704$

(ii) 95
$(95)^2 = (100 - 5)^2$
$= 100^2 + 5^2 - (2 \times 5 \times 100)$
$= 10000 + 25 - 1000$
$= 9025$

(iii) 505
$(505)^2 = (505 + 5)^2$
$= 500^2 + 5^2 + (2 \times 500 \times 5)$
$= 250000 + 25 + 5000$
$= 255025$

(iv) 702
$(702)^2 = (700 + 2)^2$
$= 700^2 + 2^2 + (2 \times 700 \times 2)$
$= 140000 + 4 + 2800$
$= 142804$

(v) 99
$(99)^2 = (100 - 1)^2$
$= 100^2 + 1^2 - (2 \times 100 \times 1)$
$= 10000 + 1 - 200$
$= 9301$

Exercise 3.4

Q1. Write the possible unit's digits of the square root of the following numbers. Which of these numbers are odd square roots?
(i) 9801
(ii) 99856
(iii) 998001
(iv) 657666025

Solution:

(i) 9801

Unit digit = 1
Unit digit of square root = 1 or 9
As number is odd, square root is also odd.

(ii) 99856
Unit digit = 6
Unit digit of square root = 4 or 6
As number is even, square root is also even.

(iii) 998001
Unit digit = 1
Unit digit of square root = 1 or 9
As number is odd, square root is also odd.

(iv) 657666025
Unit digit = 5
Unit digit of square root = 5
As number is odd, square root is also odd.

Q2. Find the square root of each of the following by prime factorization.

(i) 441	(ii) 196
(iii) 529	(iv) 1764
(v) 1156	(vi) 4096
(vii) 7056	(viii) 8281
(ix) 11664	(x) 47089
(xi) 24336	(xii) 190969
(xiii) 586756	(xiv) 27225
(xv) 3013696	

Solution:

(i) 441

$441 = 3^2 \times 7^2$

$\sqrt{441} = 3 \times 7 = 21$

3	441
3	147
7	49
7	7
	1

(ii) 196

$196 = 2^2 \times 7^2$

$\sqrt{196} = 2 \times 7 = 14$

2	196
2	98
7	49
7	7
	1

(iii) 529

$529 = 23^2$

$\sqrt{529} = 23$

23	529
23	23
	1

(iv) 1764

$1764 = 2^2 \times 3^2 \times 7^2$

$\sqrt{1764} = 2 \times 3 \times 7 = 42$

2	1764
2	882
3	441
3	147
7	49
7	7
	1

(v) 1156

$1156 = 2^2 \times 17^2$

$\sqrt{1156} = 2 \times 17 = 34$

2	1156
2	578
17	289
17	17
	1

(vi) 4096

$4096 = 2^{12}$

$\sqrt{4096} = 2^6 = 64$

2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

(vii) 7056

$7056 = 2^2 \times 2^2 \times 21^2$

$\sqrt{7056} = 2 \times 2 \times 21 = 84$

2	7056
2	3528

2	1764
2	882
21	441
21	21
	1

(viii) 8281

$8281 = 91^2$

$\sqrt{8281} = 91$

91	8281
91	91
	1

(ix) 11664

$11664 = 2^2 \times 2^2 \times 3^2 \times 3^2 \times 3^2$

$\sqrt{11664} = 2 \times 2 \times 3 \times 3 \times 3 = 108$

2	11664
2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

(x) 47089

$47089 = 7^2 \times 31^2$

$\sqrt{47089} = 7 \times 31 = 217$

7	47089
7	6727
31	961
31	31
	1

(xi) 24336

$24336 = 2^2 \times 2^2 \times 3^2 \times 13^2$

$\sqrt{24336} = 2 \times 2 \times 3 \times 13 = 156$

2	24336
2	12168
2	6084
2	3042
3	1521
3	507
13	169
13	13
	1

(xii) 190969

$190969 = 23^2 \times 19^2$

$\sqrt{190969} = 23 \times 19 = 437$

23	190969
23	8303
19	361
19	19
	1

(xiii) 586756

$586756 = 2^2 \times 383^2$

$\sqrt{586756} = 2 \times 383 = 766$

(xiv) 27225

$27225 = 5^2 \times 3^2 \times 11^2$

$\sqrt{27225} = 5 \times 3 \times 11 = 165$

(xv) 3013696

$3013696 = 2^6 \times 217^2$

$\sqrt{3013696} = 2^3 \times 217 = 1736$

2	3013696
2	1506848
2	753424
2	376712
2	188356
2	94178
217	47089
217	217
	1

Q3. Find the smallest number by which 180 must be multiplied so that it becomes a perfect square. Also, find the square root of the perfect square so obtained.

Solution:

$180 = (2 \times 2) \times (3 \times 3) \times 5$

$180 = 2^2 \times 3^2 \times 5$

To make the unpaired 5 into paired, multiply the number with 5.

Therefore, $180 \times 5 = 2^2 \times 3^2 \times 5^2$

Hence, square root of number $= \sqrt{180} \times \sqrt{5} = 2 \times 3 \times 5 = 30$

Q4. Find the smallest number by which 147 must be multiplied so that it becomes a perfect square. Also, find the square root of the number so obtained.

Solution:

$147 = 7^2 \times 3$

To make the unpaired 3 paired, multiply the number with 3.

Therefore, $147 \times 3 = 7^2 \times 3^2$

Hence, square root of number $= \sqrt{7^2} \times \sqrt{3^2} = 7 \times 3 = 21$

Q5. Find the smallest number by which 3645 must be divided so that it becomes a perfect square. Also, find the square root of the resulting number.

Solution:

$3645 = (3 \times 3) \times (3 \times 3) \times (3 \times 3) \times 5$

Here 5 is unpaired so we have to divide 3645 by 5.

Therefore,

$$\frac{3645}{5} = 3^2 \times 3^2 \times 3^2$$

Hence,

Square root of numbers $= \sqrt{\frac{3645}{5}} = 3 \times 3 \times 3 = 27$

Q6. Find the smallest number by which 1152 must be divided so that it becomes a square. Also, find the square root of the number so obtained.

Solution:

$1152 = (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times 2 \times (3 \times 3)$

Here 2 is unpaired so we have to divide 1152 with 2.

Therefore,

$$\frac{1152}{2} = 2^2 \times 2^2 \times 2^2 \times 3^2$$

Hence,

Square root of numbers $= \sqrt{\frac{1152}{2}} = 2 \times 2 \times 2 \times 3 = 24$

Q7. The product of the two numbers is 1296. If one number is 16 times the other, find the numbers.

Solution:

Let a and b be two numbers.

$a \times b = 1296$

$a = 16b$

Therefore,

$16b \times \mathrm{b} = 1296$

$b^2 = 81$

$b = 9$

Therefore, $a = 144$ and $b = 9$

Q8. A welfare association collected Rs 202500 as donation from the residents. If each paid as many rupees as there were residents, find the number of residents.

Solution:

Let total residents be a.

Therefore, each paid Rs. a

Total collection $= a(a) = a^2$

Given, total collection $= 202500$

Hence,

$a = \sqrt{202500} = \sqrt{(2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5 \times 5)} = 2 \times 3 \times 3 \times 5 \times 5$

$a = 450$

Therefore, total residents $= 450$

Q9. A society collected Rs 92.16. Each member collected as many paise as there were members. How many members were there and how much did each contribute?

Solution:

Let there were a members.

Therefore, each attributed a paise.

Therefore,

$a(a)$, i.e. total cost collected $= 9216$ paise

$a^2 = 9216$

$a = \sqrt{9216}$

$a = 2 \times 2 \times 2 \times 12$

$a = 96$

Therefore, there were 96 members and each contributed 96 paise.

Q10. A society collected Rs 2304 as fees from its students. If each student paid as many paise as there were students in the school, how many students were there in the school?

Solution:

Let a be number of school students.

Therefore, each student contributed a paise.

Total money obtained $= a^2$ paise

According to the question,

$a^2 = 230400$ paise

$a = \sqrt{230400}$

$a = \sqrt{2304} \times \sqrt{100}$

$a = 10\sqrt{2304}$

$a = 10 \times 2 \times 2 \times 12$

$a = 480$

Therefore, there were 480 students.

Q11. The area of a square field is 5184 m^2. A rectangular field, whose length is twice its breadth has its perimeter equal to the perimeter of the square field. Find the area of the rectangular field.

Solution:

Let 'a' be the side of square field.

Therefore,

$a^2 = 5184\ m^2$

$a = \sqrt{5184}\ m$

$a = 2 \times 2 \times 2 \times 9 = 72\ m$

Perimeter of square $= 4a = 288$ m

Perimeter of rectangle $= 2(l + b) = 288$ m

$2(2b + b) = 288$

$b = 48$ and $l = 96$

Area of rectangle $= 96 \times 48\ m^2 = 4608\ m^2$

Q12. Find the least square number, exactly divisible by each one of the numbers:

(i) 6,9, 15 and 20

(ii) 8,12,15 and 20

Solution:

(i) 6, 9, 15 and 20

L.C.M of given 4 numbers is 180.

$180 = 2^2 \times 3^2 \times 5$

To make it a perfect square, we have to multiply the number with 5.

Therefore,

$180 \times 5 = 2^2 \times 3^2 \times 5^2$

900 is the least square number divisible by 6,9,15 and 20.

(ii) 8, 2, 15 and 20

L.C.M of given 4 numbers is 360.

$360 = 2^2 \times 3^2 \times 2 \times 5$

To make it a perfect square, we have to multiply the number with $2 \times 5 = 10$

Therefore,

$360 \times 10 = 2^2 \times 3^2 \times 5^2 \times 2^2$

3600 is the least square number divisible by 8,12,15 and 20.

Q13. Find the square roots of 121 and 169 by the method of repeated subtraction.

Solution:

$121 - 1 = 120$

$120 - 3 = 117$
$117 - 5 = 112$
$112 - 7 = 115$
$115 - 9 = 106$
$106 - 11 = 95$
$95 - 13 = 82$
$82 - 15 = 67$
$67 - 17 = 50$
$50 - 19 = 31$
$31 - 21 = 10$
Clearly, we have performed operation 11 times.
Therefore, $\sqrt{121} = 11$
$169 - 1 = 168$
$168 - 3 = 165$
$165 - 5 = 160$
$160 - 7 = 153$
$153 - 9 = 144$
$144 - 11 = 133$
$133 - 13 = 120$
$120 - 15 = 105$
$105 - 17 = 88$
$88 - 19 = 69$
$69 - 21 = 48$
$48 - 23 = 25$
$25 - 25 = 0$
Clearly, we have performed subtraction 13 times.
Therefore, $\sqrt{169} = 13$

Q14. Write the prime factorization of the following numbers and hence find their square roots.
(i) 7744
(ii) 9604
(iii) 5929
(iv) 7056
Solution:
(i) 7744
$7744 = 2^2 \times 2^2 \times 2^2 \times 11^2$
$\sqrt{7744} = 2 \times 2 \times 2 \times 11 = 88$
(ii) 9604

$9604 = 2^2 \times 7^2 \times 7^2$

$\sqrt{9604} = 2 \times 7 \times 7 = 98$

(iii) 5929

$5929 = 11^2 \times 7^2$

$\sqrt{5929} = 11 \times 7 = 77$

(iv) 7056

$7056 = 2^2 \times 2^2 \times 7^2 \times 3^2$

$\sqrt{7056} = 2 \times 2 \times 7 \times 3 = 84$

Q15. The students of class VIII of a school donated Rs 2401 to PM's National Relief Fund. Each student donated as many rupees as the number of students in the class, Find the number of students in the class.

Solution:

Let a be the number of students.

Therefore, each student denoted a rupee.

So,

Total amount collected $= a \times a$ rupees

$= 2401$

$a^2 = 2401$

$a = 49$

Therefore, there are 49 students in the class.

Q16. A PT teacher wants to arrange a maximum possible number of 6000 students in a field such that the number of rows is equal to the number of columns. Find the number of rows if 71 were left out after arrangement.

Solution:

Let a be number of rows.

Therefore,

No. of columns = a

Total number of students who sat in field $= a^2$

Total students $= a^2 + 71 = 6000$

$a^2 = 5929$

$a = \sqrt{5929}$

$a = 11 \times 7$

$a = 77$

Therefore, total number of rows $= 77$

Exercise 3.5

Q1. Find the square root of each of the following by long division method:

(i) 12544	(ii) 97344
(iii) 286225	(iv) 390625
(v) 363609	(vi) 974169
(vii) 120409	(viii) 1471369
(ix) 291600	(x) 9653449
(xi) 1745041	(xii) 4008004
(xiii) 20657025	(xiv) 152547201
(xv) 20421361	(xvi)62504836
(xvii) 82264900	(xviii) 3226694416
(xix) 6407522209	(xx) 3915380329

Solution:

(i) 12544

	1 1 2
1	$\overline{1}\ \overline{25}\ \overline{44}$
	1
21	25
	21
222	4 44
	4 44
224	0

Therefore, $\sqrt{12544} = 112$

(ii) 97344

	3 1 2
3	$\overline{9}\ \overline{73}\ \overline{44}$
	9
61	73
	61
622	12 44
	12 44
624	0

Therefore, $\sqrt{97344} = 312$

(iii) 286225

	5 3 5
5	$\overline{28}\ \overline{62}\ \overline{25}$
	25
103	3 62
	3 09
1065	53 25
	53 25
1070	0

Therefore, $\sqrt{286225} = 535$

(iv) 390625

	6 2 5
6	$\overline{39}\ \overline{06}\ \overline{25}$
	36
122	3 06
	2 44
1245	62 25
	62 25
1250	0

Therefore, $\sqrt{390625} = 625$

(v) 363609

	6 0 3
6	$\overline{36}\ \overline{36}\ \overline{09}$
	36
120	36
	0
1203	36 09
	36 09
1206	0

Therefore, $\sqrt{363609} = 603$

(vi) 974169

	9 8 7
9	$\overline{97}\ \overline{41}\ \overline{69}$
	81
188	16 41
	15 04
1967	1 37 69
	1 37 69
1974	0

Therefore, $\sqrt{974169} = 987$

(vii) 120409

	3 4 7
3	$\overline{12}\ \overline{04}\ \overline{09}$
	9
64	3 04
	2 56
687	48 09
	48 09
694	0

Therefore, $\sqrt{120409} = 347$

(viiii) 1471369

	1 2 1 3
1	$\overline{1}\ \overline{47}\ \overline{13}\ \overline{69}$
	1
22	47
	44
241	3 13
	2 41
2423	72 69
	72 69
2426	0

Therefore, $\sqrt{1471369} = 1213$

(ix) 291600

	5 4 0
5	$\overline{29}\ \overline{16}\ \overline{00}$
	25
104	4 16
	4 16
1080	0
	0
1080	0

Therefore, $\sqrt{291600} = 540$

(x) 9653449

	3 1 0 7
3	$\overline{9}\ \overline{65}\ \overline{34}\ \overline{49}$
	9
61	65
	61
620	4 34
	0
6207	4 34 49
	4 34 49
6214	0

Therefore, $\sqrt{9653449} = 3107$

(xi) 1745041

	1 3 2 1
1	$\overline{1}\ \overline{74}\ \overline{50}\ \overline{41}$
	1
23	74
	69
262	5 50
	5 24
2641	26 41
	26 41
2642	0

Therefore, $\sqrt{1745041} = 1321$

(xii) 4008004

	2 0 0 2
2	$\overline{4}\ \overline{00}\ \overline{80}\ \overline{04}$
	4
40	0
	0
400	80
	0
4002	80 04
	80 04
4004	0

Therefore, $\sqrt{4008004} = 2002$

(xiii) 20657025

	4 5 4 5
4	$\overline{20}\ \overline{65}\ \overline{70}\ \overline{25}$
	16
85	4 65
	4 25
904	40 70
	36 16
9085	4 54 25
	4 54 25
9090	0

Therefore, $\sqrt{20657025} = 4545$

(xiv) 152547201

	1 2 3 5 1
1	$\overline{1}\ \overline{52}\ \overline{54}\ \overline{72}\ \overline{01}$
	1
22	52
	44
243	8 54
	7 29
2465	1 25 72
	1 23 25
24701	2 47 01
	2 47 01
24702	0

Therefore, $\sqrt{152547201} = 12351$

(xv) 20421361

	4 5 1 9
4	$\overline{20}\ \overline{42}\ \overline{13}\ \overline{61}$
	16
85	4 42
	4 25
901	17 13
	9 01
9029	8 12 61
	8 12 61
9038	0

Therefore, $\sqrt{20421361} = 4519$

(xvi) 62504836

	7 9 0 6
7	$\overline{62}\ \overline{50}\ \overline{48}\ \overline{36}$
	49
149	13 50
	13 41
1580	9 48
	0
15806	9 48 36
	9 48 36
15812	0

Therefore, $\sqrt{62504836} = 7906$

(xvii) 82264900

	9 0 7 0
9	$\overline{82}\ \overline{26}\ \overline{49}\ \overline{00}$
	81
180	1 26
	0
1807	1 26 49
	1 26 49
18140	0
	0
18140	0

Therefore, $\sqrt{82264900} = 9070$

(xviii) 3226694416

	5 6 8 0 4
5	$\overline{32}\ \overline{26}\ \overline{69}\ \overline{44}\ \overline{16}$
	25
106	7 26
	6 36
1128	90 69
	90 24
11360	45 44
	0
113604	45 44 16
	45 44 16
113608	0

Therefore, $\sqrt{3226694416} = 56804$

(xix) 6407522209

	8 0 0 4 7
8	$\overline{64}\ \overline{07}\ \overline{52}\ \overline{22}\ \overline{09}$
	64
160	7
	0
1600	7 52
	0
16004	7 52 22
	6 40 16
160087	1 12 06 09
	1 12 06 09
160094	0

Therefore, $\sqrt{6407522209} = 80047$

(xx) 3915380329

	6 2 5 7 3
6	39 15 38 03 29
	36
122	3 15
	2 44
1245	71 38
	62 25
12507	9 13 03
	8 75 49
125143	37 54 29
	37 54 29
125146	0

Therefore, $\sqrt{3915380329} = 62573$

Q2. Find the least number which must be subtracted from the following numbers to make them a perfect square:

(i) 2361

(ii) 194491

(iii) 26535

(iv) 161605

(v) 4401624

Solution:

(i) 2361

	4 8
4	23 61
	16
88	7 61
	7 04
96	57

Hence, 57 must be subtracted from 2361 in order to get a perfect square.

(ii) 194491

	4 4 1
4	$\overline{19}\ \overline{44}\ \overline{91}$
	16
84	3 44
	3 36
881	8 91
	8 81
882	10

Hence, 10 must be subtracted from 194491 in order to get a perfect square.

(iii) 26535

	1 6 2
1	$\overline{2}\ \overline{65}\ \overline{35}$
	1
26	1 65
	1 56
322	9 35
	6 44
324	2 91

Hence, 291 must be subtracted from 26535 in order to get a perfect square.

(iv) 161605

	4 0 2
4	$\overline{16}\ \overline{16}\ \overline{05}$
	16
80	16
	0
802	16 05
	16 04
804	1

Hence, 1 must be subtracted from 161605 in order to get a perfect square.

(v) 4401624

	2 0 9 8
2	$\overline{4}\ \overline{40}\ \overline{16}\ \overline{24}$
	4
40	40
	0
409	40 16
	36 81
4188	3 35 24
	3 35 04
4196	20

Hence, 20 must be subtracted from 4401624 in order to get a perfect square number.

Q3. Find the least number which must be added to the following numbers to make them a perfect square:

(i) 5607

(ii)4931

(iii) 4515600

(iv) 37460

(v) 506900

Solution:

(i) 5607

	7 4
7	$\overline{56}\ \overline{07}$
	49
144	7 07
	5 76
148	1 31

The remainder is 131.

Hence, $(74)^2 < 5607$

The next perfect square number is:

$(75)^2 = 5625 > 5607$

Hence, the number to be added $= 5625 - 5607 = 18$

(ii)4931

	7	0
7	$\overline{49}$	$\overline{31}$
	49	
140		31
		0
140		31

The remainder is 31.
Hence, $(70)^2 < 4931$
The next perfect square number is:
$(71)^2 = 5041 > 4931$
Hence, the number to be added $= 5041 - 4931 = 110$

(iii) 4515600

	2	1	2	4
2	$\overline{4}$	$\overline{51}$	$\overline{56}$	$\overline{00}$
	4			
41		51		
		41		
422		10	56	
		8	44	
4244		2	12	00
		1	69	76
4248			42	24

The remainder is 4224.
Hence, $(2124)^2 < 4515600$
The next perfect square number is:
$(2125)^2 = 4515625 > 4515600$
Hence, the number to be added $= 4515625 - 4515600 = 25$

(iv) 37460

	1 9 3
1	$\overline{3}\ \overline{74}\ \overline{60}$
	1
29	2 74
	2 61
383	13 60
	11 49
386	2 11

The remainder is 211.
Hence, $(193)^2 < 37460$
The next perfect square number is:
$(194)^2 = 37636 > 37460$
Hence, the number to be added $= 37636 - 37460 = 176$

(v) 506900

	7 1 1
7	$\overline{50}\ \overline{69}\ \overline{00}$
	49
141	1 69
	1 41
1421	28 00
	14 21
1422	13 79

The remainder is 1379.
Hence, $(711)^2 < 506900$
The next perfect square number is:
$(712)^2 = 506944 > 506900$
Hence, the number to be added $= 506944 - 506900 = 44$

Q4. Find the greatest number of 5 digits which is a perfect square.

Solution:
We know that:
Greatest 5-digit number $= 99999$

	3 1 6
3	$\overline{9}\ \overline{99}\ \overline{99}$
	9
61	99
	61
626	38 99
	37 56
632	1 43

The remainder is 143.
Therefore, the greatest 5-digit perfect square number is:
99999 – 143
= 99856
Hence, 99856 is the required greatest 5-digit perfect square number.

Q5. Find the least number of 4 digits which is a perfect square.

Solution:

We know that:
Least 4-digit number $= 1000$

	3 1
3	$\overline{10}\ \overline{00}$
	9
61	1 00
	61
62	39

The remainder is 39.
Therefore,
$(31)^2 < 1000$
Hence, the next perfect square number is:
$(32)^2 = 1024 > 1000$
Hence, 1024 is the required number.

Q6. Find the least number of 6-digits which is a perfect square.

Solution:

We know that:

Least 6-digit number = 100000

	3 1 6
3	$\overline{10}\ \overline{00}\ \overline{00}$
	9
61	1 00
	61
626	39 00
	37 56
632	1 44

The remainder is 144.
Therefore,
$(316)^2 < 100000$
Hence, the next perfect square is:
$(317)^2 = 100489 > 100000$
Hence, 100489 is the required number.

Q7. Find the greatest number of 4 digits which is a perfect square.

Solution:

We know that:
Greatest 4-digit number = 9999

	9 9
9	$\overline{99}\ \overline{99}$
	81
189	18 99
	17 01
198	1 98

The remainder is 198.
Hence, the greatest 4-digit perfect square number = $9999 - 198 = 9801$

Q8. A General arranges his soldiers in rows to form a perfect square. He finds that in doing so, 60 soldiers are left out. If the total number of soldiers is 8160, find the number of soldiers in each row.

Solution:

Total number of soldiers = 8160
Number of soldiers left out = 60
Number of soldiers arranged in rows to form a perfect square = $8160 - 60$
$= 8100$

Hence, number of soldiers in each row $= \sqrt{8100}$
$= \sqrt{9 \times 9 \times 10 \times 10}$
$= 90$

Q9. The area of a square field is $60025\ m^2$. A man cycles along its boundary at 18Km/hr. In how much time will he return at the starting point?

Solution:

Area of square field $= 60025\ m^2$

Speed of cyclist $= 18\ km/h = 18 \times \frac{1000}{60 \times 60} = 5\ m/s^2$

Area $= 60025\ m^2$

$Side^2 = 60025$

Side $= \sqrt{60025}$

Side $= 245$ m

Therefore, total length of boundary $= 4 \times$ Side

$= 4 \times 245$

$= 980$ m

Therefore, time taken $= \frac{980}{5} = 196$ seconds $= 3$ minutes and 16 seconds

Hence, the man will return to the starting point in 3 minutes 16 seconds.

Q10. The cost of leveling and turning a square lawn at Rs 2.50 per m^2 is Rs13322.50. Find the cost of fencing it at Rs 5 per metre.

Solution:

Rate of leveling and turning a square lawn = Rs. 2.50 per m^2

Total cost of leveling and turning = Rs. 13322.50

Total area of square lawn $= \frac{13322.50}{2.50} = 5329\ m^2$

Side of square lawn $= \sqrt{5329} = 73$ m

Total length of lawn $= 4 \times 73 = 292$ m

Cost of fencing the lawn at Rs 5 per metre $= 292 \times 5 =$ Rs. 1460

Hence, the cost of fencing the lawn is Rs 1460.

Q11. Find the greatest number of three digits which is a perfect square.

Solution:

We know that:

Largest 3-digit number = 999

	3 1
3	$\overline{9}\ \overline{99}$
	9
61	99
	61
62	38

The remainder is 38.

Hence, the greatest 3-digit perfect square number = 999 − 38 = 961

Q12. Find the smallest number which must be added to 2300 so that it becomes a perfect square.

Solution:

At first, we have to find the square root of 2300.

So, the square root of 2300 is:

	4 7
4	$\overline{23}\ \overline{00}$
	16
87	7 00
	6 09
94	91

The remainder is 91.

Hence,

$(47)^2 < 2300$

Now, the next perfect square number is $(48)^2 = 2304 > 2300$

Hence, the smallest number that must be added to 2300 to get a perfect square is:

2304 – 2300

= 4

Exercise 3.6

Q1. Find the square root of:

(i) $\frac{441}{961}$ (ii) $\frac{324}{841}$

(iii) $4\frac{29}{49}$ (iv) $2\frac{14}{25}$

(v) $2\frac{137}{196}$ (vi) $23\frac{26}{121}$

(vii) $25\frac{544}{729}$ (viii) $75\frac{46}{49}$

(ix) $3\frac{942}{2209}$ (x) $3\frac{334}{3025}$

(xi) $21\frac{2797}{3364}$ (xii) $38\frac{11}{25}$

(xiii) $23\frac{394}{729}$ (xiv) $21\frac{51}{169}$

(xv) $10\frac{151}{225}$

Solution:

(i) $\frac{441}{961}$

$$\frac{\sqrt{441}}{\sqrt{961}}=\frac{21}{31}$$

(ii) $\frac{324}{841}$

$$\frac{\sqrt{324}}{\sqrt{841}}=\frac{19}{29}$$

(iii) $\frac{225}{49}$

$$\frac{\sqrt{225}}{\sqrt{49}}=\frac{15}{7}$$

(iv) $\frac{64}{25}$

$$\frac{\sqrt{64}}{\sqrt{25}}=\frac{8}{5}$$

(v) $\frac{529}{196}$

$$\frac{\sqrt{529}}{\sqrt{196}}=\frac{23}{14}$$

(vi) $\frac{2809}{121}$

$$\frac{\sqrt{2809}}{\sqrt{121}}=\frac{53}{11}$$

(vii) $\frac{18769}{729}$

$$\frac{\sqrt{18769}}{\sqrt{729}}=\frac{137}{27}$$

(viii) $\frac{3721}{49}$

$$\frac{\sqrt{3721}}{\sqrt{49}}=\frac{61}{7}$$

(ix) $\frac{7569}{2209}$

$$\frac{\sqrt{7569}}{\sqrt{2209}}=\frac{87}{47}$$

(x) $\frac{9409}{3025}$

$$\frac{\sqrt{9409}}{\sqrt{3025}}=\frac{97}{55}$$

(xi) $\frac{73441}{3364}$

$$\frac{\sqrt{73441}}{\sqrt{3364}}=\frac{271}{58}$$

(xii) $\frac{961}{25}$

$$\frac{\sqrt{961}}{\sqrt{25}}=\frac{31}{5}$$

(xiii) $\frac{17161}{729}$

$$\frac{\sqrt{17161}}{\sqrt{729}}=\frac{131}{27}$$

(xiv) $\frac{3600}{169}$

$$\frac{\sqrt{3600}}{\sqrt{169}}=\frac{60}{13}$$

(xv) $\frac{2401}{225}$

$$\frac{\sqrt{2401}}{\sqrt{225}}=\frac{49}{15}$$

Q2. Find the value of:

(i) $\frac{\sqrt{80}}{\sqrt{405}}$

(ii) $\frac{\sqrt{441}}{\sqrt{625}}$

(iii) $\frac{\sqrt{1587}}{\sqrt{1728}}$

(iv) $\sqrt{72}\times\sqrt{338}$

(v) $\sqrt{45}\times\sqrt{20}$

Solution:

(i) $\frac{\sqrt{80}}{\sqrt{405}}=\frac{\sqrt{16}}{\sqrt{81}}$ (Cancelling numerator and denominator with 5)

$=\frac{4}{9}$ (Since , $\sqrt{16}=4, \sqrt{81}=9$)

(ii) $\frac{\sqrt{441}}{\sqrt{625}}$

$=\frac{\sqrt{16}}{\sqrt{81}}=\frac{21}{25}$ (Therefore, $\sqrt{441}=21, \sqrt{625}=25$)

(iii) $\frac{\sqrt{1587}}{\sqrt{1728}}=\frac{\sqrt{529}}{\sqrt{576}}$ (Cancelling numerator and denominator with 3)

$=\frac{23}{24}$ (Since, $\sqrt{529}=23, \sqrt{576}=24$)

(iv) $\sqrt{72} \times \sqrt{338}$

$= \sqrt{2 \times 2 \times 2 \times 3 \times 3} \times \sqrt{2 \times 13 \times 13}$

We know that,

$\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$

$\sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 13 \times 13} = 2^2 \times 3 \times 13 = 156$

(v) $\sqrt{45} \times \sqrt{20}$

$= \sqrt{3 \times 3 \times 5} \times \sqrt{5 \times 2 \times 2}$

We know that,

$\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$

$\sqrt{5 \times 5 \times 3 \times 3 \times 2 \times 2} = 5 \times 3 \times 2 = 30$

Q3. The area of a square field is $80 \frac{244}{729}$ square metres. Find the length of each side of the field.

Solution:

Given area $= 80 \times \frac{244}{729}$ m^2

$= \dfrac{58564}{729}$ m^2

If L is length of each side.

Therefore, $L^2 = \frac{58564}{729}$

$\mathrm{L} = \frac{\sqrt{58564}}{\sqrt{729}}$ (Since, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$)

$= \dfrac{242}{27}$

Q4. The area of a square field is $30 \frac{1}{4}$ m^2. Calculate the length of the side of the square.

Solution:

Given, area $= 30 \times \frac{1}{4}$ m^2

$= \dfrac{121}{4}$ m^2

If L is length of each side then,

$L^2 = \dfrac{121}{4}$

$L = \sqrt{\dfrac{121}{4}} = \dfrac{\sqrt{121}}{\sqrt{4}}$

$= \frac{11}{2}$ (Since, $\sqrt{121} = 11, \sqrt{4} = 2$)

Therefore, length is $\frac{11}{2}$ m.

Q5. Find the length of a side of a square playground whose area is equal to the area of a rectangular field of dimensions 72 m and 338 m.

Solution:

Area of rectangular field $= I \times b$

$= 72 \times 338 \text{ m}^2$

$= 24336 \text{ m}^2$

Area of square $= L^2 = 24336 \text{ m}^2$

$L = \sqrt{24336}$

$= 156 \text{ m}$

Therefore, the side length of the square playground is 156 m.

Exercise 3.7

Q1. Find the square root of the following numbers in decimal form:
84.8241

Solution:

84.8241

	9	.2	1
9	$\overline{84}$	$.\overline{82}$	$\overline{41}$
	81		
182	3	82	
	3	64	
1841		18	41
		18	41
1842			0

Therefore, $\sqrt{84.8241} = 9.21$

Q2. Find the square root of the following numbers in decimal form:
0.7225

Solution:

0.7225

	0	.8	5
0	$\overline{0}$	$.\overline{72}$	$\overline{25}$
	0		
8		72	
		64	
165		8	25
		8	25
170			0

$\sqrt{0.7225} = 0.85$

Q3. Find the square root of the following numbers in decimal form: 0.813604

Solution:

0.813604

	0	.9	0	2
0	$\overline{0}$	$.\overline{81}$	$\overline{36}$	$\overline{04}$
	0			
9		81		
		81		
180			36	
			0	
1802			36	04
			36	04
1804				0

Therefore, $\sqrt{0.813604} = 0.902$

Q4. Find the square root of the following numbers in decimal form: 0.00002025

Solution:

0.00002025

	0 .0 0 4 5
0	$\overline{0}$.$\overline{00}$ $\overline{00}$ $\overline{20}$ $\overline{25}$
	0
0	0
	0
0	0
	0
4	20
	16
85	4 25
	4 25
90	0

Therefore, $\sqrt{0.00002025} = 0.0045$

Q5. Find the square root of the following numbers in decimal form:
150.0625

Solution:

150.0625

	1 2 .2 5
1	$\overline{1}$ $\overline{50}$.$\overline{06}$ $\overline{25}$
	1
22	50
	44
242	6 06
	4 84
2445	1 22 25
	1 22 25
2450	0

Therefore, $\sqrt{150.0625} = 12.25$

Q6. Find the square root of the following numbers in decimal form:
225.6004

Solution:

225.6004

	1 5 .0 2
1	$\overline{2}$ $\overline{25}$.$\overline{60}$ $\overline{04}$
	1
25	1 25
	1 25
300	60
	0
3002	60 04
	60 04
3004	0

Therefore, $\sqrt{225.6004} = 15.02$

Q7. Find the square root of the following numbers in decimal form:
3600.720036

Solution:

3600.720036

	6 0 .0 0 6
6	$\overline{36}$ $\overline{00}$.$\overline{72}$ $\overline{00}$ $\overline{36}$
	36
120	0
	0
1200	72
	0
12000	72 00
	0
120006	72 00 36
	72 00 36
120012	0

Therefore, $\sqrt{3600.720036} = 60.006$

Q8. Find the square root of the following numbers in decimal form:
236.144689

Solution:

236.144689

	1 5 .3 6 7
1	$\overline{2}\ \overline{36}\ .\overline{14}\ \overline{46}\ \overline{89}$
	1
25	1 36
	1 25
303	11 14
	9 09
3066	2 05 46
	1 83 96
30727	21 50 89
	21 50 89
30734	0

Therefore, $\sqrt{236.144689} = 15.367$

Q9. Find the square root of the following numbers in decimal form:
0.00059049

Solution:

0.00059049

	0 .0 2 4 3
0	$\overline{0}\ .\overline{00}\ \overline{05}\ \overline{90}\ \overline{49}$
	0
0	0
	0
2	5
	4
44	1 90
	1 76
483	14 49
	14 49
486	0

Therefore, $\sqrt{0.00059049} = 0.0243$

Q10. Find the square root of the following numbers in decimal form:
176.252176

Solution:

176.252176

	1 3 .2 7 6
1	$\overline{1}\ \overline{76}\ .\overline{25}\ \overline{21}\ \overline{76}$
	1
23	76
	69
262	7 25
	5 24
2647	2 01 21
	1 85 29
26546	15 92 76
	15 92 76
26552	0

Therefore, $\sqrt{176.252176} = 13.276$

Q11. Find the square root of the following numbers in decimal form:
9998.0001

Solution:

9998.0001

	9 9 .9 9
9	$\overline{99}\ \overline{98}\ .\overline{00}\ \overline{01}$
	81
189	18 98
	17 01
1989	1 97 00
	1 79 01
19989	17 99 01
	17 99 01
19998	0

Therefore, $\sqrt{9998.0001} = 99.99$

Q12. Find the square root of the following numbers in decimal form:
0.00038809

Solution:

0.00038809

	0 .0 1 9 7
0	$\overline{0}$.$\overline{00}$ $\overline{03}$ $\overline{88}$ $\overline{09}$
	0
0	0
	0
1	3
	1
29	2 88
	2 61
387	27 09
	27 09
394	0

Therefore, $\sqrt{0.00038809} = 0.0197$

Q13. What is that fraction which when multiplied by itself gives 227.798649?

Solution:

	1 5 .0 9 3
1	$\overline{2}$ $\overline{27}$.$\overline{79}$ $\overline{86}$ $\overline{49}$
	1
25	1 27
	1 25
300	2 79
	0
3009	2 79 86
	2 70 81
30183	9 05 49
	9 05 49
30186	0

Therefore, $a = \sqrt{227.798649} = 15.093$

Q14. The area of a square playground is 256.6404 square metres. Find the length of one side of the playground.

Solution:

Given: area $= L^2 = 256.6404 \text{ m}^2$

	16.02
1	$\overline{2}\ \overline{56}\ .\overline{64}\ \overline{04}$
	1
26	1 56
	1 56
320	64
	0
3202	64 04
	64 04
3204	0

Therefore, $L = \sqrt{25.6404} = 16.02$ m

Q15. What is the fraction which when multiplied by itself gives 0.00053361?

Solution:

Let the required number be a.

Therefore, $a^2 = 0.00053361$

	0.0231
0	$\overline{0}\ .\overline{00}\ \overline{05}\ \overline{33}\ \overline{61}$
	0
0	0
	0
2	5
	4
43	1 33
	1 29
461	4 61
	4 61
462	0

Therefore, $a = 0.0231$

Q16. Simplify:

(i) $\frac{\sqrt{59.29}-\sqrt{5.29}}{\sqrt{59.29}+\sqrt{5.29}}$

(ii) $\frac{\sqrt{0.2304}+\sqrt{0.1764}}{\sqrt{0.2304}-\sqrt{0.1764}}$

Solution:

(i) $\frac{\sqrt{59.29}-\sqrt{5.29}}{\sqrt{59.29}+\sqrt{5.29}}$

At first, we find $\sqrt{59.29}$ and $\sqrt{5.29}$

Therefore, $\sqrt{59.29} = \frac{\sqrt{5929}}{\sqrt{100}} = \frac{77}{10} = 7.7$

And,

$$\sqrt{5.29} = \frac{\sqrt{529}}{\sqrt{100}} = \frac{23}{10} = 2.3$$

Now,

$$\frac{7.7 - 2.3}{7.7 + 2.3} = 0.54$$

(ii) $\frac{\sqrt{0.2304}+\sqrt{0.1764}}{\sqrt{0.2304}-\sqrt{0.1764}}$

At first, we find $\sqrt{0.2304}$ and $\sqrt{0.1764}$.

Therefore, $\sqrt{0.2304} = \frac{\sqrt{2304}}{\sqrt{10000}} = \frac{48}{100} = 0.48$

And,

$$\sqrt{0.1764} = \frac{\sqrt{0.1764}}{\sqrt{10000}} = \frac{42}{100} = 0.42$$

Now,

$$\frac{0.48 + 0.42}{0.48 - 0.42} = 15$$

Q17. Evaluate $\sqrt{50625}$ and hence find the value of $\sqrt{506.25} + \sqrt{5.0625}$.

Solution:

Given: $\sqrt{50625}$

	2 2 5
2	$\overline{5}$ $\overline{06}$ $\overline{25}$
	4
42	1 06
	84
445	22 25
	22 25
450	0

Now, $\sqrt{506.25} = \frac{\sqrt{50625}}{\sqrt{100}} = \frac{225}{10} = 22.5$

$\sqrt{5.0625} = \frac{\sqrt{50625}}{\sqrt{10000}} = \frac{225}{100} = 2.25$

$\sqrt{506.25} + \sqrt{5.0625}$

$= 22.5 + 2.25$

$= 24.75$

Q18. Find the value of $\sqrt{103.0225}$ and hence find the value of

(i) $\sqrt{10302.25}$

(ii) $\sqrt{1.030225}$

Solution:

Given: $\sqrt{103.0225}$

	1	0	.1	5
1	1	03	.02	25
	1			
20		3		
		0		
201		3	02	
		2	01	
2025		1	01	25
		1	01	25
2030				0

Now,

(i) $\sqrt{10302.25} = \sqrt{103.0225 \times 100} = 10 \times 10.15 = 101.5$

(ii) $\sqrt{1.030225} = \frac{\sqrt{103.0225}}{10} = \frac{10.15}{10} = 1.015$

Exercise 3.8

Q1. Find the square root of each of the following correct to three places of decimal.

(i) 5	(ii) 7
(iii) 17	(iv) 20
(v) 66	(vi) 427
(vii) 1.7	(viii) 23.1
(ix) 2.5	(x) 237.615
(xi) 15.3215	(xii) 0.9
(xiii) 0.1	(xiv) 0.016
(xv) 0.00064	(xvi) 0.019

(xvii) $\frac{7}{8}$ (xviii) $\frac{5}{12}$

(xix) $2\frac{1}{2}$ (xx) $287\frac{5}{8}$

Solution:

(i) 5

	2 .2 3 6
2	$\overline{5}$.$\overline{00}$ $\overline{00}$ $\overline{00}$
	4
42	1 00
	84
443	16 00
	13 29
4466	2 71 00
	2 67 96
4472	3 04

Therefore, $\sqrt{5} = 2.236$

(ii) 7

	2 .6 4 5
2	$\overline{7}$.$\overline{00}$ $\overline{00}$ $\overline{00}$
	4
46	3 00
	2 76
524	24 00
	20 96
5285	3 04 00
	2 64 25
5290	39 75

Therefore, $\sqrt{7} = 2.645$

(iii) 17

	4 .1 2 3
4	$\overline{17}\ .\overline{00}\ \overline{00}\ \overline{00}$
	16
81	1 00
	81
822	19 00
	16 44
8243	2 56 00
	2 47 29
8246	8 71

Therefore, $\sqrt{17} = 4.123$

(iv) 20

	4 .4 7 2
4	$\overline{20}\ .\overline{00}\ \overline{00}\ \overline{00}$
	16
84	4 00
	3 36
887	64 00
	62 09
8942	1 91 00
	1 78 84
8944	12 16

Therefore, $\sqrt{20} = 4.472$

(v) 66

	8 .1 2 4
8	$\overline{66}\ .\overline{00}\ \overline{00}\ \overline{00}$
	64
161	2 00
	1 61
1622	39 00
	32 44
16244	6 56 00
	6 49 76
16248	6 24

Therefore, $\sqrt{66} = 8.124$

(vi) 427

	2 0 .6 6 3
2	$\overline{4}\ \overline{27}\ .\overline{00}\ \overline{00}\ \overline{00}$
	4
40	27
	0
406	27 00
	24 36
4126	2 64 00
	2 47 56
41323	16 44 00
	12 39 69
41326	4 04 31

Therefore, $\sqrt{427} = 20.664$

(vii) 1.7

	1 .3 0 3
1	$\overline{1}\ .\overline{70}\ \overline{00}\ \overline{00}$
	1
23	70
	69
260	1 00
	0
2603	1 00 00
	78 09
2606	21 91

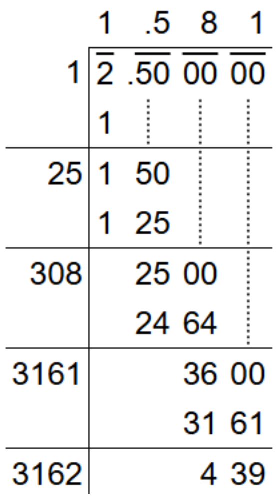
Therefore, $\sqrt{1.7} = 1.303$

(viii) 23.1

	4 .8 0 6
4	$\overline{23}\ .\overline{10}\ \overline{00}\ \overline{00}$
	16
88	7 10
	7 04
960	6 00
	0
9606	6 00 00
	5 76 36
9612	23 64

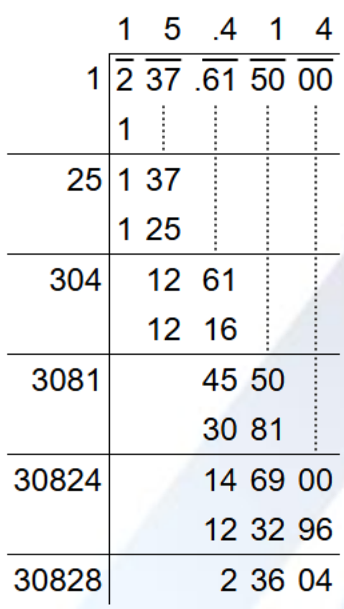
Therefore, $\sqrt{23.1} = 4.806$

(ix) 2.5



Therefore, $\sqrt{2.5} = 1.581$

(x) $237.615 = 15.415$



Therefore, $\sqrt{237.615} = 15.415$

(xi) 15.3215

	3 .9 1 4
3	$\overline{15}$.$\overline{32}$ $\overline{15}$ $\overline{00}$
	9
69	6 32
	6 21
781	11 15
	7 81
7824	3 34 00
	3 12 96
7828	21 04

Therefore, $\sqrt{15.3215} = 3.914$

(xii) 0.9

	0 .9 4 8
0	$\overline{0}$.$\overline{90}$ $\overline{00}$ $\overline{00}$
	0
9	90
	81
184	9 00
	7 36
1888	1 64 00
	1 51 04
1896	12 96

Therefore, $\sqrt{0.9} = 0.948$

(xiii) 0.1

	0 .3 1 6
0	$\overline{0}\ .\overline{10}\ \overline{00}\ \overline{00}$
	0
3	10
	9
61	1 00
	61
626	39 00
	37 56
632	1 44

Therefore, $\sqrt{0.1} = 0.316$

(xiv) 0.016

	0 .1 2 6
0	$\overline{0}\ .\overline{01}\ \overline{60}\ \overline{00}$
	0
1	1
	1
22	60
	44
246	16 00
	14 76
252	1 24

Therefore, $\sqrt{0.016} = 0.126$

(xv) 0.00064

	0 .0 2 5
0	$\overline{0}$.$\overline{00}$ $\overline{06}$ $\overline{40}$
	0
0	0
	0
2	6
	4
45	2 40
	2 25
50	15

Therefore, $\sqrt{0.00064} = 0.025$

(xvi) 0.019

	0 .1 3 7
0	$\overline{0}$.$\overline{01}$ $\overline{90}$ $\overline{00}$
	0
1	1
	1
23	90
	69
267	21 00
	18 69
274	2 31

Therefore, $\sqrt{0.019} = 0.138$

(xvii) $\frac{7}{8} = 0.875$

	0 .9 3 5
0	$\overline{0}$.$\overline{87}$ $\overline{50}$ $\overline{00}$
	0
9	87
	81
183	6 50
	5 49
1865	1 01 00
	93 25
1870	7 75

Therefore, $\sqrt{\frac{7}{8}} = 0.935$

(xviii) $\frac{5}{12} = 0.4166$

	0 .6 4 5
0	$\overline{0}$.$\overline{41}$ $\overline{66}$ $\overline{00}$
	0
6	41
	36
124	5 66
	4 96
1285	70 00
	64 25
1290	5 75

Therefore, $\sqrt{\frac{5}{12}} = 0.645$

(xix) $2\frac{1}{2} = 2.500000$

	1 .5 8 1
1	$\overline{2}$ $\overline{.50}$ $\overline{00}$ $\overline{00}$
	1
25	1 50
	1 25
308	25 00
	24 64
3161	36 00
	31 61
3162	4 39

Therefore, $\sqrt{2\frac{1}{2}} = 1.581$

(xx) $287\frac{5}{8} = 287.625$

	1 6 .9 5 9
1	$\overline{2}$ $\overline{87}$ $\overline{.62}$ $\overline{50}$ $\overline{00}$
	1
26	1 87
	1 56
329	31 62
	29 61
3385	2 01 50
	1 69 25
33909	32 25 00
	30 51 81
33918	1 73 19

Therefore, $\sqrt{287\frac{5}{8}} = 16.959$

Q2. Find the square root of 12.0068 correct to four decimal places.

Solution:

The square root of 12.0068 is:

	3 .4 6 5 0
3	$\overline{12}\ .\overline{00}\ \overline{68}\ \overline{00}\ \overline{00}$
	9
64	3 00
	2 56
686	44 68
	41 16
6925	3 52 00
	3 46 25
69300	5 75 00
	0
69300	5 75 00

Therefore, $\sqrt{12.0068} = 3.4650$

Q3. Find the square root of 11 correct to five decimal places.

Solution:

The square root of 11 is:

	3 .3 1 6 6 2
3	$\overline{11}\ .\overline{00}\ \overline{00}\ \overline{00}\ \overline{00}\ \overline{00}$
	9
63	2 00
	1 89
661	11 00
	6 61
6626	4 39 00
	3 97 56
66326	41 44 00
	39 79 56
663322	1 64 44 00
	1 32 66 44
663324	31 77 56

Hence, $\sqrt{11} = 3.31662$

Q4. Give that: $\sqrt{2} = 1.414, \sqrt{3} = 1.732, \sqrt{5} = 2.236$ and $\sqrt{7} = 2.646$, evaluate each of the following:

(i) $\sqrt{\frac{144}{7}}$

(ii) $\sqrt{\frac{2500}{3}}$

Solution:

(i) $\sqrt{\frac{144}{7}} = \frac{\sqrt{12}\times\sqrt{12}}{\sqrt{7}}$

$= \frac{12}{2.646}$

$= 4.535$

(ii) $\sqrt{\frac{2500}{3}} = \frac{\sqrt{5}\times\sqrt{5}\times\sqrt{10}\times\sqrt{10}}{\sqrt{3}}$

$= \frac{5 \times 10}{\sqrt{3}}$

$= \frac{50}{1.732}$

$= 28.867$

Q5. Given that $\sqrt{2} = 1.414, \sqrt{3} = 1.732, \sqrt{5} = 2.236$ and $\sqrt{7} = 2.646$, find the square roots of the following:

(i) $\frac{169}{75}$

(ii) $\frac{400}{63}$

(iii) $\frac{150}{7}$

(iv) $\frac{256}{5}$

(v) $\frac{276}{50}$

Solution:

(i) $\frac{169}{75}$

$$\sqrt{\frac{169}{75}} = \frac{\sqrt{13}\times\sqrt{13}}{\sqrt{5}\times\sqrt{5}\times\sqrt{3}} = \frac{13}{5\sqrt{3}} = \frac{13}{5(1.732)} = \frac{13}{8.66} = 1.50$$

(ii) $\frac{400}{63}$

$$\sqrt{\frac{400}{63}} = \frac{\sqrt{2}\times\sqrt{2\times 10\times 10}}{\sqrt{3}\times\sqrt{3}\times\sqrt{7}} = \frac{2\times 10}{3\sqrt{7}} = \frac{20}{3(2.646)} = \frac{20}{7.938} = 2.519$$

(iii) $\frac{150}{7}$

$$\sqrt{\frac{150}{7}} = \frac{\sqrt{3 \times 5 \times 5 \times 2}}{\sqrt{7}} = \frac{5\sqrt{3} \times \sqrt{2}}{\sqrt{7}} = \frac{5 \times 1.731 \times 1.414}{2.646} = \frac{12.24524}{2.646} = 4.627$$

(iv) $\frac{256}{5}$

$$\sqrt{\frac{256}{5}} = \frac{\sqrt{16} \times \sqrt{16}}{\sqrt{5}} = \frac{16}{\sqrt{5}} = \frac{16}{2.236} = 7.155$$

(v) $\frac{276}{50}$

$$\sqrt{\frac{276}{50}} = \frac{\sqrt{2 \times 2 \times 3 \times 23}}{\sqrt{5} \times \sqrt{5} \times \sqrt{2}} = \frac{2 \times \sqrt{3} \times \sqrt{23}}{5\sqrt{2}} = \frac{2 \times 1.732 \times 4.796}{5(1.414)} = 0.735$$

Exercise 3.9

Q1. Using square root table, find the square roots of the following:
7

Solution:

From square root table,

Square root of 7 is:

$\sqrt{7} = 2.645$

Therefore, the square root of 7 is 2.645.

Q2. Using square root table, find the square roots of the following:
15

Solution:

From square root table,

Square root of 15 is:

$\sqrt{15} = 3.872$

Therefore, the square root of 15 is 3.872.

Q3. Using square root table, find the square roots of the following:
74

Solution:

From square root table,

Square root of 74 is:

$\sqrt{74} = 8.602$

Therefore, the square root of 74 is 8.602.

Q4. Using square root table, find the square roots of the following:
82

Solution:

From square root table,

Square root of 82 is:

$\sqrt{82} = 9.055$

Therefore, the square root of 82 is 9.055.

Q5. Using square root table, find the square roots of the following:
198

Solution:

From square root table,

Square root of 198 is:

$\sqrt{198} = 14.071$

Therefore, the square root of 198 is 14.071.

Q6. Using square root table, find the square roots of the following:
540

Solution:

From square root table,

Square root of 540 is:

$\sqrt{540} = 23.237$

Therefore, the square root of 540 is 23.237.

Q7. Using square root table, find the square roots of the following:
8700

Solution:

From square root table,

Square root of 8700 is:

$\sqrt{8700} = 93.237$

Therefore, the square root of 8700 is 93.237.

Q8. Using square root table, find the square roots of the following:
3509

Solution:

From square root table,

Square root of 3509 is:

$\sqrt{3509} = 59.236$

Therefore, the square root of 3509 is 59.236.

Q9. Using square root table, find the square roots of the following:
6929

Solution:

From square root table,
Square root of 6929 is:
$\sqrt{6929} = 83.240$
Therefore, the square root of 6929 is 83.240.

Q10. Using square root table, find the square roots of the following:
25720

Solution:

From square root table,
Square root of 25720 is:
$\sqrt{25720} = 160.374$
Therefore, the square root of 25720 is 160.374.

Q11. Using square root table, find the square roots of the following:
1312

Solution:

From square root table,
Square root of 1312 is:
$\sqrt{1312} = 36.221$
Therefore, the square root of 1312 is 36.221.

Q12. Using square root table, find the square roots of the following:
4192

Solution:

From square root table,
Square root of 4192 is:
$\sqrt{4192} = 64.745$
Therefore, the square root of 4192 is 64.745.

Q13. Using square root table, find the square roots of the following:
49555

Solution:

From square root table,
Square root of 49555 is:

$\sqrt{49555} = 222.609$

Therefore, the square root of 49555 is 222.609.

Q14. Using square root table, find the square roots of the following:

$\frac{99}{144}$

Solution:

From square root table,

Square root of $\frac{99}{144}$ is:

$$\sqrt{\frac{99}{144}} = 0.829$$

Therefore, the square root of $\frac{99}{144}$ is 0.829.

Q15. Using square root table, find the square roots of the following:

$\frac{57}{169}$

Solution:

From square root table,

Square root of $\frac{57}{169}$ is:

$$\sqrt{\frac{57}{169}} = 0.580$$

Therefore, the square root of $\frac{57}{169}$ is 0.580.

Q16. Using square root table, find the square roots of the following:

$\frac{101}{169}$

Solution:

From square root table,

Square root of $\frac{101}{169}$ is:

$$\sqrt{\frac{101}{169}} = 0.773$$

Therefore, the square root of $\frac{101}{169}$ is 0.773.

Q17. Using square root table, find the square roots of the following:

13.21

Solution:

From square root table,

Square root of 13.21 is:

$\sqrt{13.21} = 3.634$

Therefore, the square root of 13.21 is 3.634.

Q18. Using square root table, find the square roots of the following:

21.97

Solution:

From square root table,

Square root of 21.97 is:

$\sqrt{21.97} = 4.687$

Therefore, the square root of 21.97 is 4.687.

Q19. Using square root table, find the square roots of the following:

110

Solution:

From square root table,

Square root of 110 is:

$\sqrt{110} = 10.488$

Therefore, the square root of 110 is 10.488.

Q20. Using square root table, find the square roots of the following:

1110

Solution:

From square root table,

Square root of 1110 is:

$\sqrt{1110} = 33.316$

Therefore, the square root of 1110 is 33.316.

Q21. Using square root table, find the square roots of the following:

11.11

Solution:

From square root table,

Square root of 11.11 is:

$\sqrt{11.11} = 3.333$

Therefore, the square root of 11.11 is 3.333.

Q22. The area of a square field is 325 m^2. Find the approximate length of one side of the field.

Solution:

Area of the field $= 325\ m^2$

In order to find approximate length of the side of the field we will have to calculate the square root of 325.

$\sqrt{325} = 18.027$ m

Hence, the approximate length of one side of the field is 18.027 m.

Q23. Find the length of a side of a square, whose area is equal to the area of a rectangle with sides 240 m and 70 m.

Solution:

According to the question,

Area of square = Area of rectangle

$\text{Side}^2 = 240 \times 70$

$\text{Side} = \sqrt{240 \times 70}$

$= \sqrt{10 \times 10 \times 2 \times 2 \times 2 \times 3 \times 7}$

$= 20\sqrt{42}$

$= 20 \times 6.48$

$= 129.60$ m