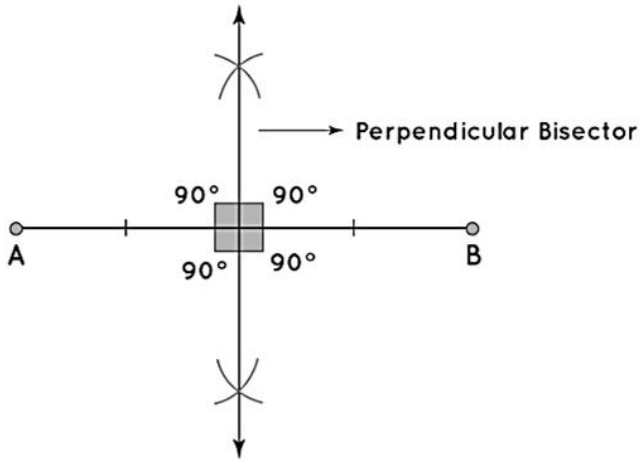
Perpendicular Bisector

Introduction

Perpendicular bisector divides a line segment exactly into two equal halves and makes an angle 90° at the intersection point. It passes through the midpoint of a line segment. We can construct it using a compass and a ruler. It makes 90° on both sides of the line segment that is being bisected. This article provides comprehensive overview of the perpendicular bisector, including its definition, construction, properties, and applications.

What is a perpendicular bisector?

A line that divides a line segment into two equal halves and forms 90° angle with the segment. In other terms, it is a line that cuts another line segment exactly in its mid-point and is perpendicular to it.



Mathematically, if a line segment has its end points as A(x_1, y_1) and B(x_2, y_2), then:

- The midpoint M of the segment is given by:
  $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
- The slope of segment AB is:
  $m = \frac{y_2 - y_1}{x_2 - x_1}$
- The slope of a perpendicular bisector is the negative reciprocal of the original slope
  $m' = -\frac{1}{m}$
- The equation of the perpendicular bisector can be found using the point-slope form of a line:
  $y - y_m = m'(x - x_m)$

How to Construct a Perpendicular Bisector?

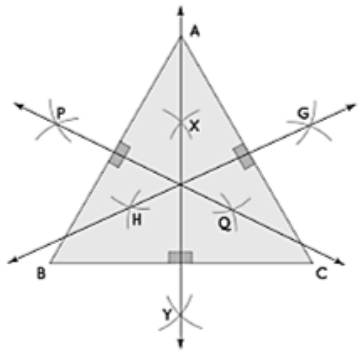
These are following steps to construct a perpendicular bisector of a line segment.

- Step 1: Draw a line segment of any suitable length.

- Step 2: Take a compass, with one endpoint as the center and with more than half of the line segment as width, draw arcs above and below the line segment.
- Step 3: Repeat the same step with another endpoint as the center.
- Step 4: Label the points of intersection.
- Step 5: Join these points. The point at which the perpendicular bisector intersects the line segment is its midpoint.

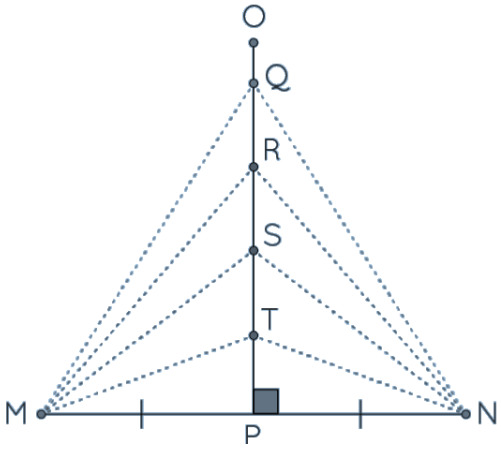
Perpendicular Bisector of a Triangle

In a triangle, a perpendicular bisector is a line that is perpendicular from the opposite vertex and bisects the side.



Perpendicular Bisector Theorem:

The perpendicular bisector theorem states that any point on the perpendicular bisector is equidistant from both the endpoints of the line segment on which it is drawn.



In the above figure, MT = NT, MS = NS, MR = NR, and MQ = NQ.

Circumcentre of a Triangle:

- The circumcentre is the point where all three perpendicular bisectors of a triangle meet.
- It is the center of the circumcircle, which passes through all three vertices of the triangle.
- The circumcentre can be inside, outside, or on the triangle, depending on whether the triangle is acute, obtuse, or right-angled, respectively.

Properties of Perpendicular Bisector

1. **Divides a line segment into two equal halves:** The perpendicular bisector of the line segment is equal to the both halves.
2. **Forms a Right Angle (90 Degrees):** It is always perpendicular to the given line segment.
3. **All Points on the Perpendicular Bisector are Equidistant:** Any point on the perpendicular bisector is equidistant from the endpoints of the original line segment.
4. **Intersection of Perpendicular Bisectors in a Triangle (Circumcenter):** The perpendicular bisectors of the three sides of a triangle meet at the circumcenter.
5. **Used in Coordinate Geometry:** The equation of the perpendicular bisector can be derived using the midpoint and the negative reciprocal of the segment's slope.

Conclusion

Perpendicular line is an important topic in geometry. It has several uses in different fields like construction, design, and navigation. Understanding the concept of perpendicularity helps us in understanding symmetry, coordinate geometry, and spatial reasoning. Deep understanding in this topic provides a basis for solving more complex mathematical problems, such as trigonometry and algebra.

Frequently Asked Questions (FAQs)

Q1. What is the difference between a median and a perpendicular bisector?
A: A perpendicular bisector is a line that is perpendicular to a segment and passes through its midpoint, while a median of a triangle is a line segment joining the midpoint of the side from the corresponding opposite vertex.

Q2. Can a perpendicular bisector pass through a vertex of a triangle?
A: Yes, in an isosceles or equilateral triangle, the perpendicular bisector of a side may pass through the opposite vertex.

Q3. How is a perpendicular bisector different from an angle bisector?
A: A perpendicular bisector divides a line segment into two equal parts at a right angle, while an angle bisector divides an angle into two equal angles.

Q4. Is the perpendicular bisector always inside the triangle?
A: No, the perpendicular bisectors of an obtuse triangle intersect outside the triangle, while for an acute triangle, they intersect inside.